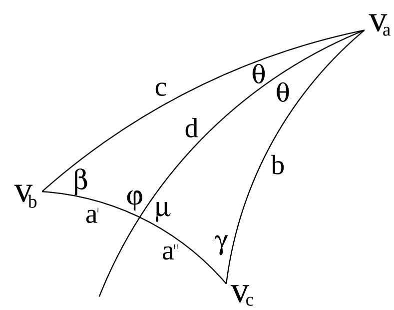
Question

Let T be a triangle in $\mathbf{H}$ with vertices v_a, v_b, and v_c. Show that the ray from v_a bisecting the angle at v_a contains the midpoint of the hyperbolic line segment ℓ joining v_b and v_c if and only if the angles at v_b and v_c are equal.

Answer



Suppose $\beta = \gamma$. Then by lcII, using side of length d implies that $\phi = \mu$. Since $\beta = \gamma$ and $\phi = \mu$, applying lcII to both subtriangle yields that

$$\begin{aligned} \cosh(a') &= \cosh(a'') \\ &= \frac{\cos(\theta) + \cos(\gamma)\cos(\mu)}{\sin(\gamma)\sin(\mu)} \end{aligned}$$

and so $a' = a''$.

Suppose now that $a' = a''$. Then, using ls, we see that $\frac{\sinh(a')}{\sin(\theta)} = \frac{\sinh(d)}{\sin(\beta)} = \frac{\sinh(d)}{\sin(\gamma)} = \frac{\sinh(a'')}{\sin(\theta)}$ and so $\beta = \gamma$, as desired.