QUESTION The length of time a customer is in a queue waiting to be served at a certain cash point has cdf $F(x) = 1 - pe^{-\lambda x}, x \geq 0, \lambda > 0, 0 < p < 1$. Find P(x=0) and the pdf for x>0. Hence find the mean and the variance of the queueing time.

ANSWER $F(x) = 1 - pe^{-\lambda x} \quad x \geq 0, \lambda > 0, 0 < p < 1$
$P(X = 0) = F(0) = 1 - p$
$f(x) = \frac{dF(x)}{dx} = \lambda pe^{-\lambda x}$
$\mu = 0 \times (1 - p) + \int_0^\infty \lambda pxe^{-\lambda x}\, dx = \frac{p}{\lambda}$ (Since $\int_0^\infty \lambda pxe^{-\lambda x}\, dx = \frac{1}{\lambda}$)
$E(X^2) = 0^2 \times (1 - p) + \int_0^\infty \lambda px^2 e^{-\lambda x} = \frac{2p}{\lambda^2}$ (Since $\int_0^\infty \lambda px^2 e^{-\lambda x}\, dx = \frac{2}{\lambda^2}$)
Therefore $\sigma^2 = \frac{2p}{\lambda^2} - \frac{p^2}{\lambda^2} = \frac{p(2-p)}{\lambda^2}$
Note that this is an example of a mixed discrete and continuous distribution.