

QUESTION A random variable X has age specific failure rate function $\lambda(x) = x$. Find $f(x)$ and $F(x)$ and derive the mode and median of the distribution of x.

ANSWER $G(x) = e^{-\int \lambda(x) dx} = e^{-\int x dx} = e^{-\frac{1}{2}x^2}$

$$f(x) = \lambda(x)G(x) = xe^{-\frac{1}{2}x^2}$$

$$F(x) = 1 - G(x) = 1 - e^{-\frac{1}{2}x^2}$$

Mode m: $f'(m) = 0, f'(x) = e^{-\frac{1}{2}x^2}$ therefore $m=1$

Median M: $F(M) = G(M) = \frac{1}{2}, e^{-\frac{1}{2}M^2} = \frac{1}{2}, \frac{1}{2}M^2 = \ln 2,$

$$M = \sqrt{2 \ln 2} = 1.177 > m$$