Vector Fields
Conservative Fields

Question
For the following vector field, find whether it is conservative. If so, find a corresponding potential
$\underline{F}(x, y, z) = y\underline{i} + x\underline{j} + z^2\underline{k}$
Answer

$$\begin{aligned} \Rightarrow \frac{\partial F_1}{\partial y} &= 1 = \frac{\partial F_2}{\partial x} \\ \frac{\partial F_1}{\partial z} &= 0 = \frac{\partial F_3}{\partial x} \\ \frac{\partial F_2}{\partial z} &= 0 = \frac{\partial F_3}{\partial y} \end{aligned}$$

So $\underline{F}$ may be conservative.
If $\underline{F} = \nabla\phi$

$$\Rightarrow \frac{\partial \phi}{\partial x} = y, \quad \frac{\partial \phi}{\partial y} = x, \quad \frac{\partial \phi}{\partial z} = z^2.$$

$$\begin{aligned} \Rightarrow \phi(x, y, z) &= \in y\, dx = xy + C_1(y, z) \\ x &= \frac{\partial \phi}{\partial y} = x + \frac{\partial C_1}{\partial y} \\ \Rightarrow \frac{\partial C_1}{\partial y} &= 0 \\ C_1(y, z) &= C_2(z), \quad \phi(x, y, z) = xy + C_2(z) \\ z^2 &= \frac{\partial \phi}{\partial z} = C_2'(z) \\ \Rightarrow C_2(z0 &= \frac{z^3}{3}. \end{aligned}$$

So $\phi(x, y, z) = xy + \frac{z^3}{3}$ is a potential for $\underline{F}$, and $\underline{F}$ is conservative on $\Re^3$.