Vector Fields
Conservative Fields

Question
If the potential ϕ is given by $\phi(\underline{r}) = \frac{1}{|\underline{r}-\underline{r}_0|^2}$, then find the corresponding three-dimensional vector field.
Answer

$$\begin{aligned}
\phi(\underline{r}) &= \frac{1}{|\underline{r}-\underline{r}_0|^2} \\
\frac{\partial\phi}{\partial x} &= -\frac{2}{|\underline{r}-\underline{r}_0|^3}\frac{(\underline{r}-\underline{r}_0)\bullet\frac{\partial \underline{r}}{\partial x}}{|\underline{r}-\underline{r}_0|} \\
&= -\frac{2(x-x_0)}{|\underline{r}-\underline{r}_0|^4}
\end{aligned}$$

Similar formulae hold for other first partials of ϕ.

$$\begin{aligned}
\Rightarrow \underline{F} &= \nabla\phi \\
&= -\frac{2}{|\underline{r}-\underline{r}_0|^4}[(x-x_0)\underline{i}+(y-y_0)\underline{j}+(z-z_0)\underline{k}] \\
&= -2\frac{\underline{r}-\underline{r}_0}{|\underline{r}-\underline{r}_0|^4}.
\end{aligned}$$

This is the vector field with scalar potential ϕ.