

QUESTION

- (a) Components produced by a certain manufacturing process have a 4% failure rate, the distribution of failures being random. To detect the failures a screening test is devised. This test picks out 95% of the failed components, but unfortunately also picks out 2% of the components which are perfectly produced.
- (i) Find the percentage of components that are picked out by the screening test.
- (ii) Determine the probability that a component which is picked out by the screening test is a faulty one.

ANSWER

- (a) From the given information,

$$p(\text{failure}) = p(F) = 0.04,$$

$$p(\text{perfect}) = p(\text{not fail}) = p(NF) = 1 - 0.04 = 0.96$$

$$p(\text{positive test}|F) = 0.95,$$

$$p(\text{positive test}|NF) = 0.02$$

- (i) $p(\text{positive test})$

$$= p(\text{positive test}|F)p(F) + p(\text{positive test}|NF)p(NF)$$

$$= (0.95)(0.04) + (0.02)(0.96) = 0.0572$$

i.e. the test picks out 5.72% of components.

- (ii) We want

$$\begin{aligned} p(F|\text{pos. test}) &= \frac{p(F \text{ and pos. test})}{p(\text{pos. test})} \\ &= \frac{p(\text{pos. test}|F)p(F)}{p(\text{pos. test})} \\ &= \frac{(0.95)(0.04)}{(0.0572)} \\ &= 0.664 \end{aligned}$$