QUESTION

Determine whether or not the following sequence is convergent

$a_n = \dfrac{1 + n(-1)^n}{n}$ for $n = 1, 2, \ldots$

ANSWER

$a_n = \dfrac{1 + n(-1)^n}{n} = \dfrac{1}{n} + (-1)^n$. As $n \to \infty$ the $\dfrac{1}{n}$ term tends to 0 but

$(-1)^n = \begin{cases} +1 & n \text{ even} \\ -1 & n \text{ odd} \end{cases}$

Thus the terms of the sequence alternate between $+1$ and $-1$, approximately, so the sequence is not convergent.