Multiple Integration
Iteration of Double Integrals

Question

Find the volume for the solid defined by

The space inside the cylinder $x^2 + 2y^2 = 8$ that is over the plane $z = y - 4$
and under the plane $z = 8 - x$.

Answer

The part of $z = 8 - x$ that lies inside $x^2 = 2y^2 = 8$ lies over $z = 0$. The part
of $z = y - 4$ that lies inside the cylinder lies under $z = 0$.

$$
\begin{aligned}
\Rightarrow V &= \iint_{x^2+2y^2\leq 8} (8 - x - (y-4))\, dA \\
&= \iint_{x^2+2y^2\leq 8} 12\, dA \\
&= 12 \times (\text{area of } \frac{x^2}{8} + \frac{y^2}{4} = 1) \\
&= 12 \times \pi(2\sqrt{2})(2) \\
&= 48\sqrt{2}\pi \text{cu. units}
\end{aligned}
$$