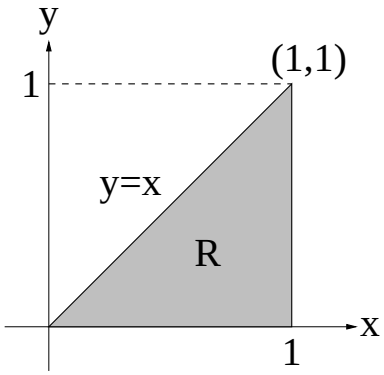
Multiple Integration
Iteration of Double Integrals

Question
Sketch the domain of integration, and calculate the iterated integral for

$$\int_0^1 dy \int_y^1 e^{-x^2}\, dx$$

Answer



$$\begin{aligned} I &= \int_0^1 dy \int_y^1 e^{-x^2}\, dx \\ &= \int_R e^{-x^2}\, dx \\ &= \int_0^1 e^{-x^2} \int_0^x dy \\ &= \int_0^1 x e^{-x^2}\, dx \\ \text{Let } u &= x^2 \\ \Rightarrow du &= 2x dx \end{aligned}$$

$$\begin{aligned} \Rightarrow I &= \frac{1}{2}\int_0^1 e^{-u} \\ &= -\frac{1}{2}e^{-u}\Big|_0^1 \\ &= \frac{1}{2}\left(1 - \frac{1}{e}\right) \end{aligned}$$