QUESTION Use the complex inversion formula to find

$$\mathcal{L}^{-1}\frac{s}{(s-2)^3}$$

ANSWER

$$f(s) = \frac{s}{(s-2)^3}$$

There is a triple pole at $s = 2$, let $u = s - 2$

$$\begin{aligned}
e^{st}f(s) &= e^{2t}\frac{(u+2)e^{ut}}{u^3} \\
&= \frac{e^{2t}}{u^3}(u+2)\left(1 + ut + \frac{u^2t^2}{2} + \ldots\right) \\
&= \frac{e^{2t}}{u^3}(u+2)\left(1 + ut + \frac{u^2t^2}{2} + \ldots\right) \\
&= \frac{e^{2t}}{u^3}\left(2 + (1+2t)u + \left(\frac{t^2}{2} + t\right)u^2 + \ldots\right) \\
F(t) &= \text{Res}\left(e^{st}f(s), s = 2\right) = e^{2t}\left(\frac{t^2}{2} + t\right)
\end{aligned}$$