Question

(a) Four non-coplanar points A, B, C, D lie at equal distances from the point O. A point P is defined by

$$2\vec{OP} = \vec{OA} + \vec{OB} + \vec{OC} + \vec{OD}.$$

show that the line joining P to the mid point of any edge of the tetrahedron $ABCD$ is at right angles to the opposite edge.

(b) Suppose that O is the centre of a circle $A - 1A_2A_3$ of unit radius in a plane π. If the points $B_1, , B_2, B_3$ are defined by $\vec{OB_1} = \vec{OA_2} + \vec{OA_3}$, $\vec{OB_2} = \vec{OA_3} + \vec{OA_1}$, $\vec{OB_3} = \vec{OA_1} + \vec{OA_2}$ show that B_1, B_2, B_3 are the centres of other unit circles in π which pass through A_2 and A_3, A_3 and A_1, A_1 and A_1 respectively.

Prove that the three circles, centers B_i meet at C where $\vec{OC} =)\vec{A_1} + \vec{OA_2} + \vec{OA_3}$.

Answer

(a) The midpoint M of CD is $\frac{1}{2}(\vec{OC} + \vec{OD})$.

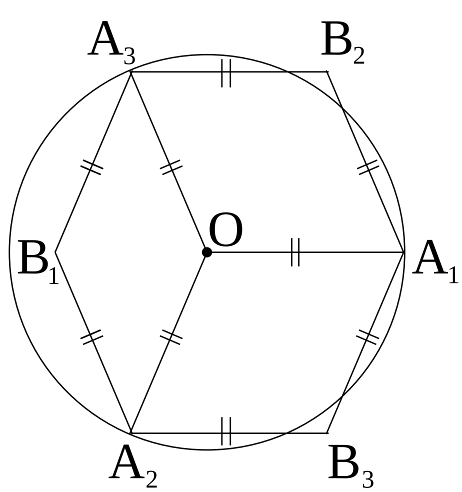
The point P is $\frac{1}{2}(\vec{OA} + \vec{OB} + \vec{OC} + \vec{OD}$

So $\vec{MP} = \frac{1}{2}(\vec{OA} + \vec{OB})$

The direction of the opposite edge $\vec{AB} = \vec{OB} - \vec{OA}$

So $\vec{MP} \cdot \vec{AB} = \frac{1}{2}(|OB|^2 - |OA|^2) = 0$

(b)



first part os obvoius from parallelograms.

$\vec{B_1C} = \vec{OC} - \vec{OB_1} = \vec{OC} - (\vec{OA_2} + \vec{OA_3}) = \vec{OA_1}$
So $|B_1C| = 1$
Similarly $|B_2C| = 1$ and $|B_3C| = 1$
So C lies on all three circles.