QUESTION

Let $C$ denote the boundary (taken in counterclockwise sense) of the square whose sides lie along the lines $x = \pm 2$ and $y = \pm 2$. Evaluate the following integrals:

**(a)**

$$\int_C \frac{e^{-z}}{z - (\pi i/2)} dz$$

**(b)**

$$\int_C \frac{cosz}{z(z^2+8)} dz$$

**(c)**

$$\int_C \frac{coshz}{z^2} dz$$

ANSWER

We shall use the Cauchy integral formulae in the following form.

$$\int_c \frac{f(z)dz}{(z-b)^{n+1}} = \frac{2\pi i f^{(n)}(b)}{n!} \qquad (*)$$

Here $f$ is analytic within and on a closed contour $c$.

**(a)** $C$ is the square bounded by $x = \pm 2, y = \pm 2$ so $\pi i/2$ lies within $C$. Thus by (*) with $n = 0$ we get $\int_C \frac{e^{-z}dz}{z-\pi/2} = 2\pi i e^{-\pi i/2} = 2\pi i(-i) = 2\pi$.

**(b)** As $\sqrt{8}$ lies outside $C$ we let $f(z) = \frac{\cos z}{z^2+8}$ in (*). Then

$$\int_C \frac{\cos z}{z^2+8} = 2\pi i f(o) = 2\pi i/8 = \pi i/4.$$

**(c)** In (*) we now put $n = 1,\ f(z) = \cosh z$ so

$$\int_C \frac{\cosh z}{z^2} = 2\pi i \sinh 0 = 0.$$