Ordinary Differential Equations
Classification

Question
Show that $y = x$ is a solution of $y'' + y = x$.
Find a solution y to satisfy $y(\pi) = 1$ and $y'(\pi) = 0$.
Answer
If $y = y_1(x) = x$ then this will give $y_1' = 1$ and $y_1'' = 0$. Thus

$$y_1'' + y_1 = 0 + x.$$

$y_2 = A\cos x + B\sin x$ is a solution of $y'' + y = 0$ and so

$$y = y_1(x) + y_2(x) = x + A\cos x + B\sin x$$

is also a solution.
The solution will satisfy

$$\begin{aligned} 1 &= y(\pi) = \pi - A \\ 0 &= y'(\pi) = 1 - B \end{aligned}$$

if A and B take the values

$$\begin{aligned} A &= \pi - 1 \\ B &= 1 \end{aligned}$$

So the solution is

$$y = x + (\pi - 1)\cos x + \sin x$$