Ordinary Differential Equations
Classification

Question
Show that $y = \cos x$ and $y = \sin x$ are solutions of $y'' + y = 0$.
Which of the following are solutions? Justify your answer.

(a) $\sin x - \cos x$

(b) $\sin(x + 3)$

(c) $\sin 2x$

Answer

$$\begin{aligned} \text{If } y &= \cos x \\ \Rightarrow y'' + y &= -\cos x + \cos x = 0 \\ \text{If } y &= \sin x \\ \Rightarrow y'' + y &= -\sin x + \sin x = 0 \end{aligned}$$

So $y = \cos x$ and $y = \sin x$ are both solutions.
As the DE is linear and homogeneous, any function of the form

$$y = A\cos x + B\sin x$$

is also a solution.

(a) $\sin x - \cos x$ fits with $A = -1$, $B = 1$ and so is a solution.

(b) $\sin(x + 3) = \sin 3 \cos x + \cos 3 \sin x$ fits with $A = \sin 3$, $B = \cos 3$ and so is also a solution.

(c) $\sin 2x$ cannot be represented in the form $A\cos x + B\sin x$ and therefore is not a solution.