Question
A particle of mass m moves along a space curve given by
$\mathbf{r} = a\cos\omega t\mathbf{i} + b\sin\omega t\mathbf{j}$.
Find

(a) the torque about the origin of the force acting upon it,

(b) the angular momentum of the particle about the origin.

Answer
$\mathbf{r} = a\cos\omega t\mathbf{i} + b\sin\omega t\mathbf{j}$
$\dot{\mathbf{r}} = -a\omega\sin\omega t\mathbf{i} + b\omega\cos\omega t\mathbf{j}$
$\ddot{\mathbf{r}} = -\omega^2(a\cos\omega t\mathbf{i} + b\sin\omega t\mathbf{j}) = -\omega^2\mathbf{r}$

(a) Using Newton's 2nd law: $m\ddot{\mathbf{r}} = \mathbf{F}$

The torque is $\mathbf{r}\times\mathbf{F} = \mathbf{r}\times m\ddot{\mathbf{r}} = -m\omega^2\mathbf{r}\times\mathbf{r} = 0$

(b)

$$\begin{aligned}
\text{Angular momentum} &= \mathbf{r}\times m\dot{\mathbf{r}} \\
&= m\omega\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a\cos\omega t & b\sin\omega t & 0 \\ -a\sin\omega t & b\cot\omega t & 0 \end{vmatrix} \\
&= mab\omega\mathbf{k}
\end{aligned}$$