

Question

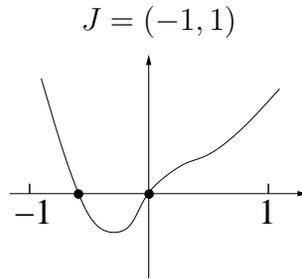
Which of the following functions ϕ are reparametrizations from the open interval

$J = (-1, 1)$ to the open interval $I = \phi(J)$?

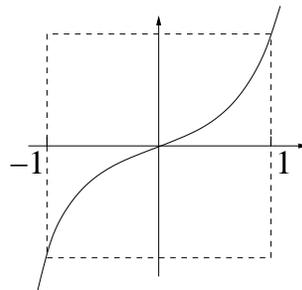
$$\phi(t) = \quad (i) \ t + 2t^4 \quad (ii) \ t + t^5 \quad (iii) \ t^3 + \sin^3 \pi t.$$

Answer

(i) **NO: not injective** (e.g. $\phi(0) = 0$ and $\phi(-\frac{1}{\sqrt[3]{2}}) = 0$, $-1 < -\frac{1}{\sqrt[3]{2}} < 0 < 1$).



(ii) **YES:** $\phi'(t) = 1 + 5t^4 > 0, \forall t$



(iii) **NO:** $\phi'(0) = 0$ so if $\psi = \phi^{-1}$ then ψ could not be differentiable at $0 = \phi(0)$.