Question

(a) Write down a formula for the sum of then odd numbers between 1 and $2n-1$ inclusive.

(b) The first term in a convergent geometric sequence is 1 and the total sum of the series is 2. What is the second term of the sequence?

Answer

(a) Arithmetic series:

$u_1 = 1, \quad u_2 = 3, \quad u_m = 1 + 2(m-1), \quad u_n = 1 + 2(n-1) = 2n - 1$
Hence there are n terms in the series.

Use the formula for summing arithmetic series:

$S_n = \frac{1}{2}n(u_1 + u_n) = \frac{1}{2}n(2n - 1 + 1) = \frac{1}{2}n \times 2n = n^2$

(b) The sum of a geometric series with constant r and first term 1 is:

$$S = \sum_{n=1}^{\infty} u_n = 1 + r + r^2 + r^3 + ... = \frac{1}{(1-r)}$$

$$S = \frac{1}{(1-r)} = 2$$

Hence $1 - r = \frac{1}{2} \Rightarrow r = \frac{1}{2} \Rightarrow u_2 = r = \frac{1}{2}$