Question

From the relations

$$\begin{aligned}
\mathbf{e}_r &= \mathbf{i}\sin\theta\cos\phi + \mathbf{j}\sin\theta\sin\phi + \mathbf{k}\cos\theta \\
\mathbf{e}_\phi &= -\mathbf{i}\sin\phi + \mathbf{j}\cos\phi \\
\mathbf{e}_\theta &= \mathbf{i}\cos\theta\cos\phi + \mathbf{j}\cos\theta\sin\phi - \mathbf{k}\sin\theta
\end{aligned}$$

derive

$$\begin{aligned}
\mathbf{i} &= \mathbf{e}_r\sin\theta\cos\phi - \mathbf{e}_\phi\sin\phi + \mathbf{e}_\theta\cos\theta\cos\phi \\
\mathbf{j} &= \mathbf{e}_r\sin\theta\sin\phi + \mathbf{e}_\phi\cos\phi + \mathbf{e}_\theta\cos\theta\sin\phi \\
\mathbf{k} &= \mathbf{e}_r\cos\theta - \mathbf{e}_\theta\sin\theta
\end{aligned}$$

Answer

$$\begin{aligned}
\mathbf{i} &= (\mathbf{i}\cdot\mathbf{e}_r)\mathbf{e}_r + (\mathbf{i}\cdot\mathbf{e}_\phi)\mathbf{e}_\phi + (\mathbf{i}\cdot\mathbf{e}_\theta)\mathbf{e}_\theta \\
&= \sin\theta\cos\phi\,\mathbf{e}_r - \sin\phi\,\mathbf{e}_\phi + \cos\theta\cos\phi\,\mathbf{e}_\theta
\end{aligned}$$

From the given expressions for $\mathbf{e}_r$, $\mathbf{e}_\theta$, $\mathbf{e}_\phi$ in terms of $\mathbf{i}, \mathbf{j}, \mathbf{k}$, $\mathbf{j}$ and $\mathbf{k}$ can be found by an analogous procedure.