Question
Calculate $\arg(z)$ defined on $[0, 2\pi)$ for the following values of z:

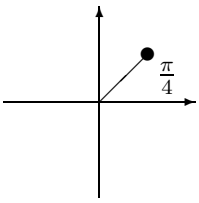
$$(i)z = 0,\ (ii)z = 1 + i,\ (iii)z = i,\ (iv)z = -1 + i$$

$$(v)z = -1.\ (vi)z = -1 - i,\ (vii)z = -i,\ (viii)z = 1 - i$$

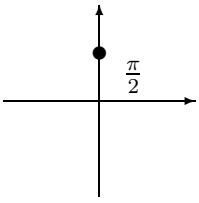
Answer
$\log z = \log|z| + i\arg(z) \quad 0 \leq \arg(z) < 2\pi$

(i) $\log(0) = \underbrace{\log|0|}_{-\infty} + i\underbrace{\arg(0)}_{\text{not defined}}$

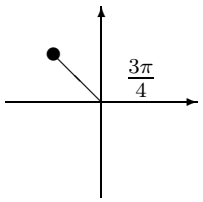
(ii) $\log(1+i) = \log|1+i| + i\arg(1+i) = \underline{\sqrt{2} + \frac{i\pi}{4}}$



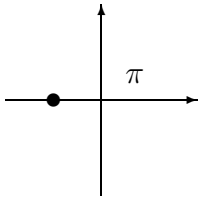
(iii) $\log(i) = \log|i| + i\arg(i) = \underline{0 + \frac{i\pi}{2}}$



(iv) $\log(-1+i) = \log|-1+i| + i\arg(-1+i) = \underline{\sqrt{2} + \frac{3i\pi}{4}}$

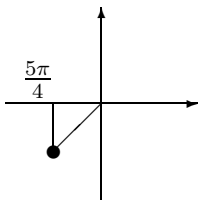


(v) $\log(-1) = \log|-1| + i\arg(-1) = 0 + i\pi = \underline{i\pi}$



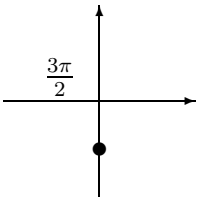
(vi) $\log(-1-i) = \log|-1-i| + i\arg(-1-i) = \underline{\log\sqrt{2} + \frac{5i\pi}{4}}$

NB Not $-\frac{3\pi}{4}i$ as before since we're now defining arg on $0 \leq \arg(z) < 2\pi$



(vii) $\log(-i) = \log|-i| + i\arg(-i) = 0 + i\dfrac{3\pi}{2} = \underline{\dfrac{3i\pi}{2}}$

NB Not $-\dfrac{i\pi}{2}$ as before since we're now defining arg on $0 \leq \arg(z) < 2\pi$



(viii) $\log(1-i) = \log|1-i| + i\arg(1-i) = \underline{\log\sqrt{2} + \dfrac{7i\pi}{4}}$

NB Not $-\dfrac{i\pi}{4}$ as before since we're now defining arg on $0 \leq \arg(z) < 2\pi$