QUESTION

Prove that if $f(n)$ is a multiplicative function such that $f(n) \neq 0$ for at least one value of n, then $f(1) = 1$.

ANSWER

Suppose $f(n) \neq 0$. Now $\gcd(1, n) = 1$, so by the multiplicative property, $f(n.1) = f(n).f(1)$, i.e. $f(n) = f(n)f(1)$, so as $f(n) \neq 0$, we may conclude $f(1) = 1$.