QUESTION From the definitions of $\sin z$ and $\cos z$ in terms of the exponential function show that $\frac{d}{dz}(\sin z) = \cos z$ and $\frac{d}{dz}(\cos z) = -\sin z$.

ANSWER $\frac{d}{dz}(\sin z) = \frac{d}{dz}\frac{e^{iz}-e^{-iz}}{2i} = \frac{i(e^{iz}+e^{-iz})}{2i} = \frac{e^{iz}+e^{-iz}}{2} = \cos z$ and similarly $\frac{d}{dz}(\cos z) = -\sin z$.