QUESTION
Assuming an interest rate of r and discrete annual payments, what is the present value of a sum of money to be recieved in T years' time, i.e., how much would you invest now to obtain the desired sum in T years? Hence what is the present value of £100 to be recieved in 10 years time when the interest rate is 5% and

(a) annual discounting is used;

(b) semi-annual discounting is used;

(c) continuous discounting.

ANSWER
Rate= r, t years, F =amount to be received in future, P=value of that money now.
It always helps to turn the question around: How much would I have to invest now to receive F in T years.

(a) Annual compounding: If I invest P now, in T years I have

$$P(1+r)^T$$

Thus if this has to be F we have

$$F + P(1+r)^T$$

or

$$P = \frac{F}{(1+r)^T}$$

the present value of F.

(b) Semiannual compounding. Follows the same argument as (a), although the interest factor is now $\left(1+\frac{r}{2}\right)^{2T}$ $(m=2)$. Thus

$$P = \frac{F}{\left(1+\frac{r}{2}\right)^{2t}}$$

(c) Continuous discounting: use $Pe^{rT} = F$

$$\Rightarrow P = Fe^{-rT}$$

Thus if $F = 100,\ r = 0.05,\ T = 10$

$$P = \begin{cases} (a) & \frac{100}{(1.05)^10} & = & 61.39 \\ (b) & \frac{100}{(1.025)^20} & = & 61.03 \\ (c) & \frac{100}{e^{-0.5}} & = & 60.65 \end{cases}$$