Question
Solve the equation

$$z^2 = -5 + 2i.$$

Express the solution in the form $a + ib, \;\; a, b \epsilon \mathbf{R}$

Answer
$z^2 = -5 + 2i.\ a, b \epsilon \mathbf{R}$

$a^2 - b^2 = -5 \Rightarrow 2ab = 2 \Rightarrow b = \dfrac{1}{a}$

$a^2 - \dfrac{1}{a^2} = -5 \Rightarrow a^4 + 5a^2 - 1 = 0 \Rightarrow a^2 = \dfrac{-5 \pm \sqrt{29}}{2}$ and $a \epsilon \mathbf{R}$

So $a^2 = \dfrac{-5 + \sqrt{29}}{2} \Rightarrow a = \pm\sqrt{\dfrac{\sqrt{29} - 5}{2}} \Rightarrow b = \pm\sqrt{\dfrac{\sqrt{29} + 5}{2}}$

$$z = \left(\sqrt{\frac{\sqrt{29} - 5}{2}} + i\sqrt{\frac{\sqrt{29} + 5}{2}}\right)$$