QUESTION

(a) Let $f(z) = (z+1)/(z-1)$. Find the Taylor series for $f(z)$ that is valid in the disc $|z| < 1$.

(b) find a Laurent series for $f(z)$ that is valid in the annular domain $1 < |z| < \infty$.

ANSWER

(a) Write $f(z) = -(1+z)(1-z)^{-1}$. Thus we have

$$f(z) = -(1+z)(1+z+z^2+\cdots) = -(1+2z+2z^2+2z^3+\cdots).$$

This is valid if $|z| < 1$.

(b) Now write $f(z) = (1+\frac{1}{z})(1-\frac{1}{z})^{-1}$ Thus we have

$$f(z) = (1+\frac{1}{z})(1+z^{-1}+z^{-2}+\cdots) = (1+2z^{-1}+2z^{-2}+\cdots).$$

This is valid if $|z| > 1$.