Exam Question
Topic: Tangent Plane
Find the equation of the tangent plane at the point $(-1, 1, 0)$ to the surface given by the implicit equation

$$xy + yz + \cos(zx) = 0.$$

Solution Let $F(x, y, z) = xy + yz + \cos(zx)$. Differentiating gives

$$\begin{aligned}
\frac{\partial F}{\partial x} &= y - z\sin(zx) = 1 \text{ at } (-1, 1, 0) \\
\frac{\partial F}{\partial y} &= x + z = -1 \text{ at } (-1, 1, 0) \\
\frac{\partial F}{\partial z} &= y + x\sin(zx) = 1 \text{ at } (-1, 1, 0)
\end{aligned}$$

So the equation of the tangent plane is $(x+1)-(y-1)+z = 0$; $x-y+z = -2$.