Exam Question
Topic: Tangent Plane
Find an equation for the plane which is tangent to the surface whose equation is $z = \ln(x^2 + y^2)$ at the point $(1, 0, 0)$.
This tangent plane meets the surface in many other points. Find all such points for which $x = 2$.

Solution

$$z = \ln(x^2 + y^2); \quad \frac{\partial z}{\partial x} = \frac{2x}{x^2 + y^2}; \quad \frac{\partial z}{\partial y} = \frac{2y}{x^2 + y^2}.$$

So when $x = 1$ and $y = 0$, $\dfrac{\partial z}{\partial x} = 2$ and $\frac{\partial z}{\partial y} = 0$.
The equation of the tangent plane at $(1, 0, 0)$ is therefore $z = 2(x - 1)$.
This plane meets the surface where $2(x - 1) = \ln(x^2 + y^2)$, so if $x = 2$ we have $2 = ln(4 + y^2)$.
Thus $4 + y^2 = \mathrm{e}^2$ i.e., $y = \pm\sqrt{\mathrm{e}^2 - 4}$.
There are therefore two such points, namely $(2, \pm\sqrt{\mathrm{e}^2 - 4}, 2)$.