COMP6250
Social Media and Network Science

Game Theory Intro

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The narrative

- Game basics
- Basic games - predicting behaviour when individuals interact
- Predicting behaviour spread and evolution in a group (next session)
- Predicting behaviour spread in a network (next sessions)

The narrative

Game basics

What is a Game

- Individuals can act according to their *self-interest* when presented with *choices*
- But when more than one individuals interact with each other their *choices* can lead to different outcomes
- Acting according to *self interest* does not always yield the maximum profit in such cases
- ***How can we reason about behaviour?***
- ***How can we predict outcomes?***

Presentation or Exam?

- You and your partner need to work on your common project and your exam at the same time
- You need to make a choice between the two
- Your grades will be determined based on how well you do on both

		Your Partner	
		Presentation	*Exam*
You	*Presentation*	90, 90	86, 92
	Exam	92, 86	88, 88

Figure 6.1: Exam or Presentation?

SOURCE: http://www.cs.cornell.edu/home/kleinber/networks-book

What is a Game

- A game is the environment where such interactions take place and it consists of:
  - A set of participants: *players*
  - Options per participant: *strategies*
  - Benefit per choice of option: *payoff*
    - Payoffs can be based on the choices not of one participant but of all participants
    - They are shown in a *payoff matrix*

Prisoner's Dilemma

- Two have been taken prisoners and are questioned by the police
- They are both guilty
- When questioned they are offered the option to confess
  - Should both of them confess they will be convicted to serve in prison for 5 years
  - Should just one of them confess, the confessor will be let free, while the other one will serve 10 years
  - Should none of them confess, they will both serve a year for resisting arrest.
- Prisoners cannot communicate with each other

Prisoner's Dilemma

2

	Confess Strategy	Not Confess Strategy
1 Confess Strategy	-5, -5	0, -10
1 Not Confess Strategy	-10, 0	-1, -1

What would you do?

Prisoner's Dilemma

2

1

	Confess Strategy (T)	Not Confess Strategy (T')
Confess Strategy (S)	$P_1(S, T), P_2(S, T)$	$P_1(S, T'), P_2(S, T')$
Not Confess Strategy (S')	$P_1(S', T), P_2(S', T)$	$P_1(S', T'), P_2(S', T')$

Best responses

- Let's assume we have a player 1 and a player 2 with strategies S and T respectively.
  - $P_1(S, T)$ and $P_2(S, T)$ are the payoffs for each player given their strategies.
- For a player, a best response is the best choice they can make given a certain expectation of a choice from the other player
- Given a choice of a strategy T by player 2, a *best response for player 1* is strategy S, when for every other available strategy S'
  - $P_1(S, T) \geq P_1(S', T)$

Strictly best responses

- Given a choice of a strategy T by player 2, a *strict best response* for player 1 is strategy S, when for every other available strategy S'
  - $P_1(S, T) > P_1(S', T)$

Dominant Strategies

- A *dominant strategy* S for Player 1 is one that is the *best response* to every strategy of Player 2.
- A *strictly dominant strategy* S for Player 1 is one that is the *strictly best response* to every strategy of Player 2.
- There is the assumption that players have *come common knowledge* of possible payoffs of each other, etc.

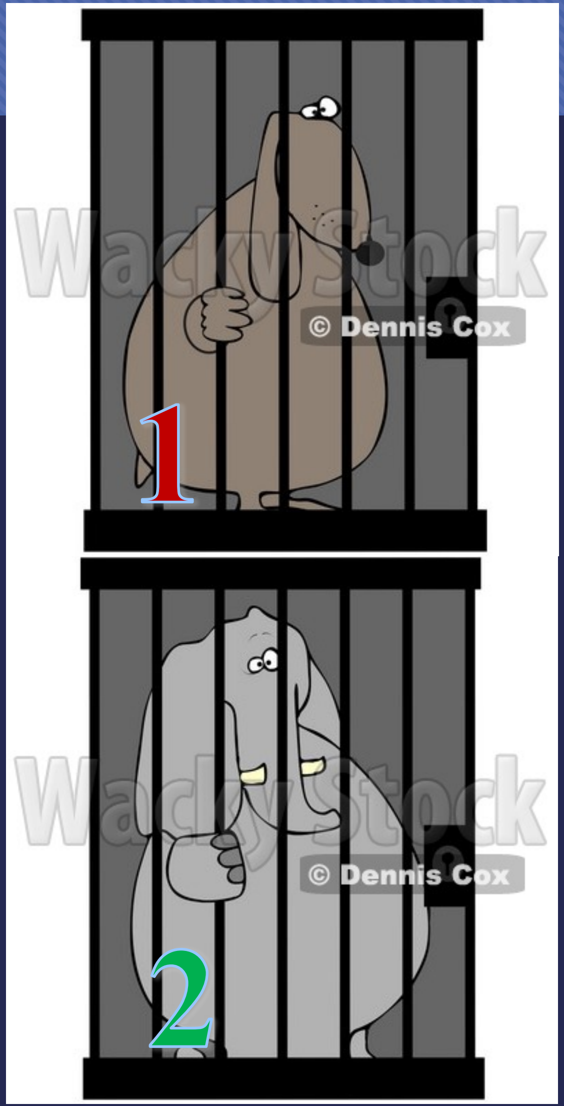
Prisoner's Dilemma

Dominant strategy for 2 **2**

Dominant strategy for 1 **1**

	Confess Strategy	Not Confess Strategy
Confess Strategy	-5, -5	0, -10
Not Confess Strategy	-10, 0	-1, -1

Best outcome for both is out of their dominant strategies



The narrative

Predicting behaviour when individuals interact

Predicting outcomes

- In games with strictly dominant strategies, we expect players to chose those strategies
  - This basic assumption has been debated but it is a basic one in game theory
- In games without strictly dominant strategies, how can we predict the choices of the players? – SEE EQUILIBRIA

Dominant strategy for one party only

Marketing strategies of a big firm (1) and smaller firm (2) for low-priced vs. upscale product launch.

The behavior of the party with the dominant strategy can be predicted.

… based on that, the behavior of the other party can be predicted too.

Firm 2

Firm 1

	Low-priced	Upscale
Low-priced	.48, .12	.60, .40
Upscale	.40, .60	.32, .08

SOURCE: http://www.cs.cornell.edu/home/kleinber/networks-book

Example - equilibria

- Firm 1 and Firm 2 are competing for clients A, B and C
- Firm 1 too small, Firm 2 is large
- They need to decide which client to approach
  - If they approach the same client they get half the client's business each
  - If Firm 1 approaches a client on its own they will get 0 business
  - If Firm 2 approaches B or C on its own, they will get their full business
  - A is a large client and will do business only with both of them and they payoff will be higher (4 each)
  - Business with B or C is worth 2

		Firm 2		
		A	B	C
Firm 1	A	4, 4	0, 2	0, 2
	B	0, 0	1, 1	0, 2
	C	0, 0	0, 2	1, 1

Figure 6.6: Three-Client Game

SOURCE: http://www.cs.cornell.edu/home/kleinber/networks-book

Example - equilibria

- (A, A) is the only Nash Equilibrium

		Firm 2		
		A	*B*	*C*
Firm 1	*A*	4, 4	0, 2	0, 2
	B	0, 0	1, 1	0, 2
	C	0, 0	0, 2	1, 1

Figure 6.6: Three-Client Game

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Nash Equilibrium

- In a game where player 1 choses strategy S and player 2 choses strategy T, the pair of strategies (S, T) is a *Nash Equilibrium* if
  - S is a best response to T, and
  - T is a best response to S.
- The expectation is that even when there are no dominant strategies, if there are Nash equilibria, players will choose the strategies of the equilibria
- This is based on the belief that each party will make this choice
- But how can we predict behaviour when there are more than one Nash Equilibria in a game?
  - And they yield the same payoffs?

Is there a Nash equilibrium in the prisoner's dilemma game?

Multiple Equilibria – Coordination games

- A Balanced Coordination Game
  - What can you and your partner choose when preparing a common presentation? Keynote or PowerPoint?
    - We assume that you cannot convert from one to the other

Two Nash Equilibria:
(P, P) (K, K)

		Your Partner	
		PowerPoint	*Keynote*
You	*PowerPoint*	1, 1	0, 0
	Keynote	0, 0	1, 1

Figure 6.7: Coordination Game

SOURCE: http://www.cs.cornell.edu/home/kleinber/networks-book

Multiple Equilibria: Focal Points

To predict which of the multiple equilibria players will chose one can argue that there can be "natural reasons" not shown in the payoff matrix that will create a bias for one equilibrium

- This will be a *focal point*
- E.g. if PowerPoint is more frequently used in the University maybe both players will chose this instead of Keynote

Reference: Schelling, T. (1960) A Strategy of Conflict. Harvard University Press

Multiple equilibria: "Battle of the sexes" game

- An unbalanced coordination game.
- Two equilibria but gains differ for each player depending on equilibrium.
- Hard to predict choice of strategies based on external conventions.

2

1		Rom-com	Action
	Rom-com	1, 2	0, 0
	Action	0, 0	2, 1

Multiple equilibria: Stag hunt game

- An unbalanced coordination game.
- The party that goes for the highest payoff gets penalised more than the party that goes for lower payoff.
- Difficult to predict behavior.
- Similar to prisoner's dilemma but what is the difference?

1 \ 2	Hunt Stag	Hunt Hare
Hunt Stag	4, 4	0, 3
Hunt Hare	3, 0	3, 3

Multiple Equilibria – Anti-coordination games

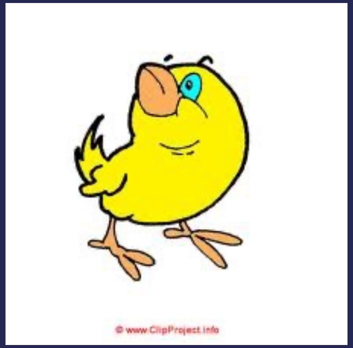
- Anti-coordination games:
  - Hawk-Dove Game
  - Chicken

2

1

	Dove strategy	Hawk Strategy
Dove Strategy	3, 3	1, 5
Hawk Strategy	5, 1	0, 0

1



Matching Pennies

- What about games with no (pure strategy) Nash Equilibria?
- Two players hold a penny each and they decide which side to show to each other each time
- Player 1 looses her/his penny if they match
- Player 2 looses his/her penny if they don't match

2

1

	Head Strategy	Tail Strategy
Head Strategy	-1, +1	+1, -1
Tail Strategy	+1, -1	-1, +1

Mixed Strategies

- When there are no equilibria (as in the matching pennies game) we can assign a probability on each strategy
  - E.g. Player 1 will choose Head with a probability *p*
    - and Tail with with probability *1-p*
    - Player 1 is choosing a *pure strategy Head* if p=1

Mixed Strategies and Equilibria

- An equilibrium with mixed strategies is one where probabilities of strategies for Player 1 is the best response to a probability of strategies by Player 2
- In the matching pennies game, we have an equilibrium for probability ½ for each strategy for each player
  - In cases where payoffs are less ‘symmetric’ equilibria are based on unequal probabilities

Strategy Optimisation

- Pure strategies vs. Mixed strategies
  - Mixed strategies can help find additional Nash equilibria or the only Nash equilibria
- Individual optimisation vs. group optimisation
  - Dominant strategies, Nash equilibria, focal points refer to individual optimisation
  - Pareto optimality and social optimality refer to group optimisation

Pareto Optimality

- **Take a choice of strategies**; it is Pareto-optimal if there is no other choice in which all players receive payoffs that
  - are *at least as high*, and
  - At least one player receives a *strictly higher* payoff
- It could be that a unique nash equilibrium is not pareto-optimal; a binding agreement is required to ensure that a pareto-optimal set of strategies is chosen in that case

Which pairs of strategies are pareto-optimal?

	Confess	Not Confess
Confess	X -5, -5	V 0, -10
Not Confess	-10, 0 V	-1, -1 V

Social Optimality

- A choice of strategies by the players that maximizes the sum of the players' payoffs
- If a pair of strategies is socially optimal is also Pareto-optimal
  - Discuss: why?
- Of, course, adding payoffs to establish social welfare has to be *meaningful*

Which pair of strategies here is socially-optimal?

		Your Partner	
		Presentation	*Exam*
You	*Presentation*	90, 90	86, 92
	Exam	92, 86	88, 88

Figure 6.1: Exam or Presentation?

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Social Optimality

- **Take a choice of strategies**; it is Pareto-optimal if there is no other choice in which all players receive payoffs that
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	Confess	Not Confess
Confess	X -5, -5	X 0, -10
Not Confess	-10, 0 X	-1, -1 V

Which pairs of strategies are socially-optimal?

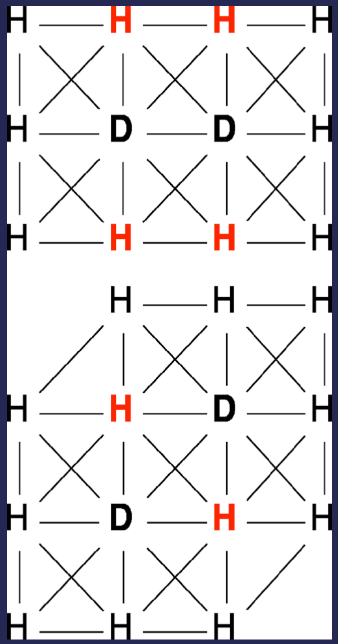
Multiplayer Games

- They can be used to model games with more than one players
- Nash equilibrium in a multiplayer game with players 1, …, n
  - A set of strategies (S_1, S_2, …, S_n) in which each strategy is the best response to all the others
  - For player *i*, strategy S_i is a best response if for any other available strategy S'_i
  - $P_i(S_1, \ldots, S_i, S_{i+1}, \ldots, S_n) \geq P_i(S_1, \ldots, S'_i, S_{i+1}, \ldots, S_n)$

Research Case

	H	D
H	$\left(\frac{G-C}{2},\frac{G-C}{2}\right)$	$(G,0)$
D	$(0,G)$	$\left(\frac{G}{2},\frac{G}{2}\right)$

- Hawks and Doves in small-world networks
- "The role of network clustering on cooperation in the Hawk-Dove game"
- Assuming static network structures
- "Dovelike behaviour is advantaged if synchronous update is used"



SOURCE: Tomassini et al. Hawks and Doves on small-world networks. Physical Review E (2006) vol. 73 (1) pp. 016132

FIG. 8. (Color online) Lattice: two possible configurations.

Two-person interaction

- Bargaining games
  - E.g. Two parties A and B bargaining how to split $1
  - A and B have outside options x and y respectively (options if they leave the negotiation)
  - The Nash bargaining solution is that the surplus ($1 – x – y) will be split between A and B
- Perceived status makes a difference in bargaining games (see Easley and Kleinberg §12.5)
- Actual behaviour is not always ‘rational’ (see ultimatum game)

The Ultimatum Game

- Person A is given **$1** to split with person B.
- B can only accept or reject the split.
- If B accepts each person gets amount proposed by A.
    - If B rejects no party gets anything.

The Ultimatum Game

- What is the rational approach for A?
- What is the rational approach for B?
- If A gives an ultimatum of a $0.99 vs. $0.01 split, should B accept?
- If A gives an ultimatum of a $1.00 vs. $0.00 split, should B accept?

The Ultimatum Game

- Discuss differences in behaviour of B depending on whether it is a human or a computer program.
- Research by Güth, Schmittberger and Schwarze show that people do behave differently (i.e. A offered on average 1/3 of the balance – in many cases A offered an even split).
- Discuss differences in behaviour of A if B is a computer program.

Werner Güth, Rolf Schmittberger, Bernd Schwarze (1982) An experimental analysis of ultimatum bargaining, Journal of Economic Behavior & Organization, Volume 3, Issue 4, Pages 367-388, ISSN 0167-2681, https://doi.org/10.1016/0167-2681(82)90011-7.

Predicting behaviour with Game Theory

- Are there (strictly) dominant strategies?
- Or any (pure) Nash equilibria?
- If there are many Nash equilibria can we predict which one will be achieved based on higher payoffs or focal points?
- What are Nash equilibria for mixed strategies?
- Are there focal points or other conventions?

- Are there pareto-optimal pairs of strategies?
  - Are Nash equilibria among them? A binding agreement would be required if not.
- Is there a socially-optimal pair of strategies?

Lessons learned

- Understanding of the main concepts of Game Theory. Given a payoff matrix be able to identify and explain best responses, dominant strategies, equilibria, focal points, pareto optimality, social optimality.
- Ability to explain how and under what circumstances Game Theory can help predict behaviour.
- **Home study material: Sections 6.1-6.9 from the Easley and Kleinberg book.**

- Easley, D. and Kleinberg, J. Networks Crowds and Markets. Cambridge University Press, 2010. http://www.cs.cornell.edu/home/kleinber/networks-book (chapters 6 and 7)