Question
A function $f : \mathbf{R^n} \rightarrow \mathbf{R}$ is said to be measurable iff

i) for all $c \epsilon \mathbf{R}$ $\{x|f(x) \leq c\}$ is measurable.

show that statement (i) is equivalent to each of the statements below

ii) for all $c \epsilon \mathbf{R}$ $\{x|f(x) < c\}$ is measurable.

iii) for all $c \epsilon \mathbf{R}$ $\{x|f(x) \geq c\}$ is measurable.

iv) for all $c \epsilon \mathbf{R}$ $\{x|f(x) > c\}$ is measurable.

Answer

$$\{x|f(x) < c\} = \bigcup_{n=1}^{\infty} \{x|f(x) \leq c - \frac{1}{n}\}$$

$$\{x|f(x) \geq c\} = \mathbf{R^n} - \{x|f(x) < c\}$$

$$\{x|f(x) > c\} = \bigcap_{n=1}^{\infty} \{x|f(x) \geq c + \frac{1}{n}\}$$