QUESTION

(a) Show that $f(z) = |z|^2$ is not analytic anywhere.

(b) Show that $f(z) = e^z$ is analytic everywhere.

ANSWER

(a) $z = x + iy$, $|z|^2 = x^2 + y^2$, $u = x^2$, $v = y^2$
$u_x = 2x \neq v_y = 2y$ Hence not analytic.

(b) $e^z = e^{x+iy} = e^x(\cos y + i \sin y)$, $u = e^x \cos y$, $v = e^x \sin y$
$u_x = e^x \cos y = v_y$, $u_y = -e^x \sin y = -v_x$

The partial derivatives are continuous and hence the function is analytic.