Question

Sketch each of the following regions and define them by inequalities of the form

$$f_1(x) \le y \le f_2(x)$$

$$a_1 \le x \le a_2$$

where f_1, f_2 are functions of x and a_1, a_2 are real constants, and also of the form
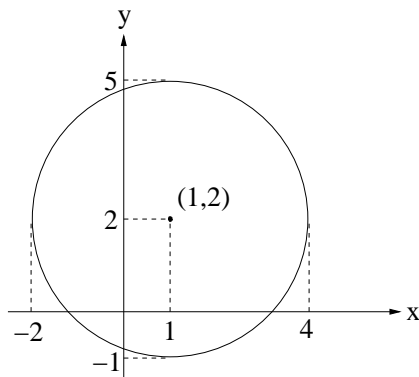
$$g_1(y) \le x \le g_2(y)$$

$$b_1 \le y \le b_2$$

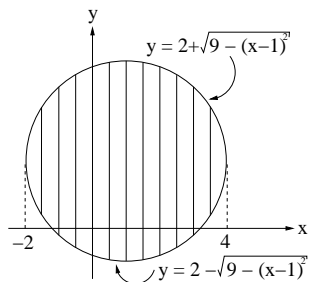where g_1, g_2 are functions of y and b_1, b_2 are real constants:

(i) the circle with centre $(1, 2)$ and radius 3;

(ii) the triangle with vertices $(1, 1)$, $(4, 1)$ and $(4, 7)$;

(iii) the region defined by the inequalities $y \le 4 - x^2$ and $y \ge (2 - x)^2$.
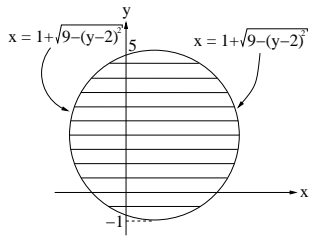
Answer

(i)

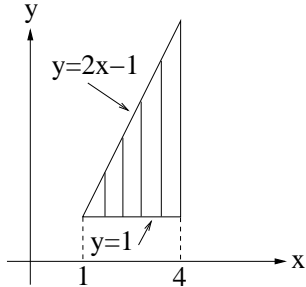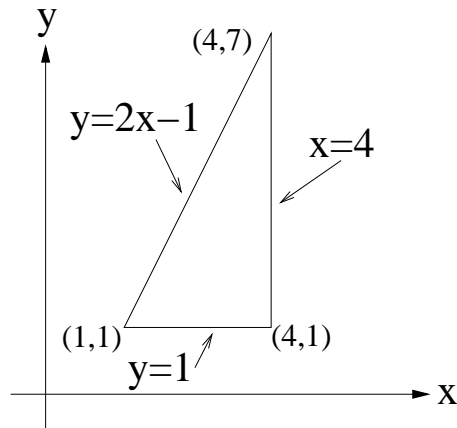The circle has equation
$$(x - 1)^2 + (y - 2)^2 = 3^2 = 9$$

Think of vertical lines (x-fixed) and let y vary from bottom to top:
$$2 - \sqrt{9 - (x - 1)^2} \le y \le 2 + \sqrt{9 - (x - 1)^2}$$
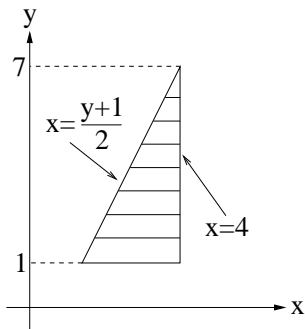Then let the line move from left to right:
$$-2 \le x \le 4$$

1

Think of horizontal lines (y-fixed)
and let x vary from left to right:
$$1 - \sqrt{9 - (y-2)^2} \leq x \leq 1 + \sqrt{9 - (y-2)^2}$$
Then let the line move from bottom to top:
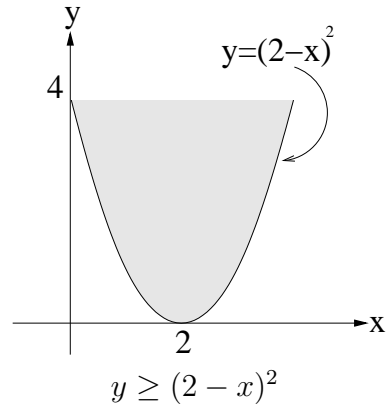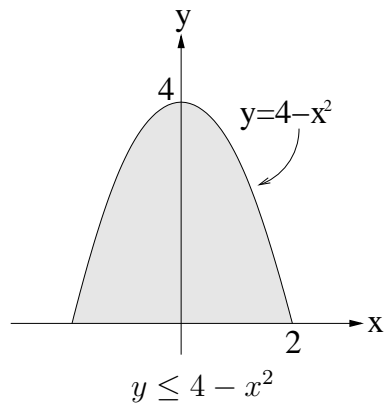$$-1 \leq y \leq 5$$

(ii)

Think of vertical lines
and let y vary from bottom to top:
$$1 \leq y \leq 2x - 1$$
Then let the line move from left to right:
$$1 \leq x \leq 4$$

Think of horizontal lines
and let x vary from left to right:
$$\frac{y+1}{2} \leq x \leq 4$$
Then let the line move from bottom to top:
$$1 \leq y \leq 7$$

(iii)

2

y

4

y=4−x²

2

x

$y \le 4 - x^2$

y

4

y=(2−x)²

2

x

$y \ge (2 - x)^2$

The two inequalities above
define the region $\longrightarrow$

y

4

y=4−x²

y=(2−x)²

2

x

Vertical lines

$y=4-x^2$

$y=(2-x)^2$

2

$(2-x)^2 \le y \le 4-x^2$
$0 \le x \le 2$

Horizontal lines

$x = \sqrt{4-y}$

4

$x = 2-\sqrt{y}$

$2 - \sqrt{y} \le x \le \sqrt{4-y}$
$0 \le y \le 4$