QUESTION A box contains 12 balls numbered from 1 to 12. The balls numbered 1 to 5 are red, those numbered 6 to 9 are white and the remaining three balls are blue. Three balls are to be drawn out at random without replacement from the box. Let A denote the event that each number drawn will be even, B the event that no blue ball will be drawn and C the event that one ball of each colour will be drawn. Calculate

(i) $P(A)$

(ii) $P(B)$

(iii) $P(C)$

(iv) $P(A \cap C)$

(v) $P(B \cup C)$

(vi) $P(A \cup B)$

ANSWER

(i) $A = \{\text{all even}\}$

$$\begin{aligned} P(A) &= \frac{\begin{pmatrix} 6 \\ 3 \end{pmatrix}}{\begin{pmatrix} 12 \\ 3 \end{pmatrix}} \\ &= \frac{6}{12} \times \frac{5}{11} \times \frac{4}{10} = \frac{1}{11} \end{aligned}$$

(ii) $B = \{\text{no blue ball}\}$

$$\begin{aligned} P(B) &= \frac{\begin{pmatrix} 9 \\ 3 \end{pmatrix}}{\begin{pmatrix} 12 \\ 3 \end{pmatrix}} \\ &= \frac{9}{12} \times \frac{8}{11} \times \frac{7}{10} = \frac{21}{55} \end{aligned}$$

(iii) $C = \{\text{one of each colour}\}$

$$\begin{aligned} P(C) &= \frac{\binom{5}{1}\binom{4}{1}\binom{3}{1}}{\binom{12}{3}} \\ &= \frac{5}{12} \times \frac{4}{11} \times \frac{3}{10} \times 3! = \frac{3}{11} \end{aligned}$$

(iv)

$$\begin{aligned} P(A \cap C) &= P(\text{all even} \cap \text{ one of each colour}) \\ &= \frac{\binom{2}{1}\binom{2}{1}\binom{2}{1}}{\binom{12}{3}} \\ &= \frac{2}{12} \times \frac{2}{11} \times \frac{2}{10} \times 3! = \frac{2}{55} \end{aligned}$$

(v) $P(B \cup C) = P(B) + P(C) - P(B \cap C)$ by addition theorem.$P(B \cap C) = P(\text{no blue} \cap \text{one of each colour} = 0$
therefore$P(B \cup C) = P(B) + P(C) = \frac{36}{55}$

(vi) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ by addition theorem.$P(A \cap B) = P(\text{all even} \cap \text{no blue} = \frac{4}{12} \times \frac{3}{11} \times \frac{2}{10} = \frac{1}{55}$
therefore$P(A \cup B) = \frac{1}{11} + \frac{21}{55} - \frac{1}{55} = \frac{36}{55}$