Question

(i) $\int 3x^2\,dx$

(ii) $\int (x^3 - 3x^2 + 8x + 7)\,dx$

(iii) $\int_1^3 (x^2 + 3x)\,dx$

(iv) $\int_0^2 (x^2 - 2x)\,dx$

(v) $\int_2^3 (x^2 - 2x)\,dx$

(vi) $\int_0^3 (x^2 - 2x)\,dx$

Answer

(i)

$\int 3x^2\,dx = x^3 + c$ (standard power rule)

(ii)

$\int (x^3 - 3x^2 + 8x + 7)\,dx = \frac{x^2}{4} - x^3 + \frac{8x^2}{2} + 7x + c = \frac{x^4}{4} - x^3 + 4x^2 + 7x + c$
(standard power rule)

(iii)

$$\begin{aligned}\int_1^3 (x^2 + 3x)\,dx &= \left[\frac{x^3}{3} + \frac{3x^2}{2}\right]_1^3 \\ &= \left(\frac{(3)^3}{3} + \frac{3(3)^2}{2}\right) - \left(\frac{1}{3} + \frac{3}{2}\right) = \underline{20\frac{2}{3}}\end{aligned}$$

(iv)

$$\begin{aligned}\int_0^2 (x^2 - 2x)\,dx &= \left[\frac{x^3}{8} - \frac{2x^2}{2}\right]_0^2 \\ &= \left[\frac{x^3}{3} - x^2\right]_0^2 \\ &= \left(\frac{8}{3} - 4\right) - (0 - 0) = \underline{-1\frac{1}{3}}\end{aligned}$$

(v)

$$\begin{aligned}\int_2^3 (x^2 - 2x)\,dx &= \left[\frac{x^3}{3} - x^2\right]_2^3 \\ &= \left(\frac{27}{3} - 9\right) - \left(\frac{8}{3} - 4\right) = \underline{1\frac{1}{3}}\end{aligned}$$

(vi)

$$\int_0^3 (x^2 - 2x)\,dx = \int_0^2 (x^2 - 2x) + \int_2^3 (x^2 - 2x) = (iv) + (v) = \underline{0}$$

(by addition property of integrals)

or longhand:

$$\int_0^3 (x^2 - 2x)\,dx = \left[\frac{x^3}{3} - x^3\right]_0^3 = \left(\frac{27}{3} - 9\right) - (0 - 0) = \underline{0}$$

PICTURE