Exam Question
Topic: Double Integral
By changing the variables so that the region R is transformed into a square,
evaluate the integral

$$\iint_R \left(5(y - x^2)^4 + 6xy\right)\left(y + 2x^2\right) d(x, y),$$

where R is the region in the first quadrant bounded by the curves

$$y = x^2, \quad y = x^2 + 1, \quad xy = 1, \quad xy = 2.$$

Solution
Let $u = y - x^2; \quad v = xy.$

$$\left|\frac{\partial(u, v)}{\partial(x, y)}\right| = \left|\begin{pmatrix} -2x & 1 \\ y & x \end{pmatrix}\right| = \left|y + 2x^2\right| = y + 2x^2 \quad \text{in the first quadrant.}$$

Therefore

$$\begin{aligned}
I &= \int_1^2 dv \int_0^1 (5u^4 + 6v) \, du = \int_1^2 \left[u^5 + 6uv\right]_{u=0}^1 dv \\
&= \int_1^2 (1 + 6v) \, dv = \left[v + 3v^2\right]_1^2 = 10.
\end{aligned}$$