Exam Question
Topic: Volume of Revolution
A region R in the x-y plane is bounded by the x-axis, the line $x = \pi/2$ and the curve $y = \sin x$.
Find the volumes generated when the region R is rotated through a complete revolution about the lines

$$\text{(i) } y = 0, \quad \text{(ii) } y = -1, \quad \text{(iii) } x = 0, \quad \text{(iv) } x = -1.$$

(Note that in parts (ii) and (iv) you should be able to use the values of integrals found in parts (i) and (iii) respectively, thereby saving some calculation. You may quote the result that the area of R is equal to 1.]

Solution
(i) $V = \pi \int_0^{\pi/2} \sin^2 x \; dx = \pi \int_0^{\pi/2} \frac{1}{2}(1 - \cos 2x) dx$
$= \pi \left[\frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right)\right]_0^{\pi/2} = \pi \times \frac{\pi}{4} = \frac{\pi^2}{4}.$
(ii) $V = \pi \int_0^{\pi/2} (1+\sin x)^2 \; dx = \pi \int_0^{\pi/2} (1+2\sin x+\sin^2 x) \; dx - \frac{\pi^2}{2} = \pi\left(2 + \frac{\pi}{4}\right)$
(iii) $V = \int_0^{\pi/2} 2\pi x \sin x \; dx = 2\pi \left[x \cos x + \sin x\right]_0^{\pi/2} = 2\pi$
(iv) $V = \int_0^{\pi/2} 2\pi(1 + x) \sin x \; dx = \int_0^{\pi/2} 2\pi \sin x \; dx + \int_0^{\pi/2} 2\pi x \sin x \; dx = 2\pi + 2\pi = 4\pi.$