Question
Find the polar equation of the ellipse

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

with the origin at a focus. Find the polar equation with the origin at the other focus.

Answer

$$\frac{x^2}{9} + \frac{y^2}{4} = 1 \qquad \text{“}a = 3,\ b = 2\text{”}$$

So $1 - e^2 = \frac{4}{9} \Rightarrow e^2 = \frac{5}{9} \Rightarrow e = \frac{\sqrt{5}}{3}$. The Foci are at $(\pm\sqrt{5}, 0)$
When $x = \sqrt{5}$ we have $\frac{5}{9} + \frac{y^2}{4} = 1 \Rightarrow y = \pm\frac{4}{3}$
So $l = \frac{4}{3}$
Thus the polar equation is

$$\frac{4}{3r} = 1 - \frac{\sqrt{5}}{3}\cos\theta$$

Referred to the other focus the equation is

$$\frac{4}{3r} = 1 + \frac{\sqrt{5}}{3}\cos\theta$$