Question

(a) Solve the following system of equations

$$\begin{aligned} x + 2y + 2z &= 11 \\ 2x - y + z &= 3 \\ -4x + 7y + z &= 13 \end{aligned}$$

Give a geometrical interpretation.

(b) Write down a 3x3 matrix which represents a transformation in 3-dimensional space consisting of a rotation of 60° about the z-axis together with a magnification in the z-direction by a scale factor of 2. Write down its inverse and check your answer by multiplication.

Answer

(a)
$$\left(\begin{array}{ccc|c} 1 & 2 & 2 & 11 \\ 2 & -1 & 1 & 3 \\ -4 & 7 & 1 & 13 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 2 & 2 & 11 \\ 0 & -5 & -3 & -19 \\ 0 & 15 & 9 & 57 \end{array}\right)$$

So $\begin{aligned} x + 2y + 2z &= 11 \\ 5y + 3z &= 19 \end{aligned}$

Let $z = t$ then $y = \dfrac{19 - 3t}{5}$

Thus $x = 11 - 2t - 2\left(\dfrac{19 - 3t}{5}\right) = \dfrac{55 - 10t - 38 + 6t}{5} = \dfrac{17 - 4t}{5}$

So $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{17}{5} \\ \frac{19}{5} \\ 0 \end{pmatrix} + t \begin{pmatrix} \frac{-4}{5} \\ \frac{-3}{5} \\ 1 \end{pmatrix}$

This system represents three plans meeting in a common line, whose equation is the solution.

(b) $A = \begin{pmatrix} \frac{1}{2} & \frac{-\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 2 \end{pmatrix}$

$A^{-1} = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ \frac{-\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} \end{pmatrix}$

Check $AA^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$