**Question**

A certain river floods every year. Suppose that the low water mark is set at 1 and the high water mark $X$ has the cdf

$$F(x) = P(X \leq x) = 1 - \frac{1}{x^2}, \ 1 \leq x < \infty.$$

**(a)** Find $f(x)$, the pdf of $X$.

**(b)** If the low water mark is reset at zero and we use a unit of measurement which is $\frac{1}{10}$ of that given previously, the high-water mark becomes $Y = 10(X-1)$. Find the cdf of $Y$.

**(c)** Find the pdf of $Y$ by differentiating the cdf as obtained in part (b), and also using the transformation technique.

**Answer**

**(a)**

$$f(x) = \begin{cases} 0 & \text{if } -\infty < x < 1 \\ \frac{2}{x^3} & \text{if } 1 \leq x < \infty. \end{cases}$$

**(b)** The cdf of $Y$ is

$$\begin{aligned} & G(y) = P\{Y \leq y\} \\ = \ & P\{10(X-1) \leq y\} \\ = \ & P\{X \leq 1 + \frac{y}{10}\} \\ = \ & F\left(1 + \frac{y}{10}\right) \\ = \ & 1 - \frac{1}{\left(1 + \frac{y}{10}\right)^2}, \quad 0 \leq y < \infty. \end{aligned}$$

**(c)** The pdf of $Y$ is

$$\begin{aligned} g(y) &= \frac{dG(y)}{dy} \\ &= 2 \cdot \frac{1}{\left(1 + \frac{y}{10}\right)^3} \frac{1}{10}, \quad 0 \leq y < \infty \\ &= \frac{1}{5} \frac{1}{\left(1 + \frac{y}{10}\right)^3}, \quad 0 \leq y < \infty \end{aligned}$$

Using the transformation technique:

The range of $y$ is $0 \leq y < \infty$.

The transformation is increasing.

$x = 1 + \frac{y}{10} = s(y)$

$\frac{dx}{dy} = \frac{1}{10}$

$$\begin{aligned} g(y) &= f(s(y)) \left| \frac{dx}{dy} \right| \\ &= \frac{2}{\left(1 + \frac{y}{10}\right)^3} \cdot \frac{1}{10} \\ &= \frac{1}{5} \frac{1}{\left(1 + \frac{y}{10}\right)^3}, \quad 0 \leq y < \infty \end{aligned}$$