QUESTION A quadratic polynomial $p(x)$ in x is frequently written in the form $p(x) = ax^2 + bx + c$. Sometimes, however, it is more convenient to write it in the form

$$p(x) = \alpha x^{(2)} + \beta x^{(1)} + \gamma x^{(0)}$$

where $x^{(2)} = x(x-1), x^{(1)} = x, x^{(0)} = 1$.
For example, $5x^2 - 8x + 2 = 5x(x-1) - 3x + 2 = 5x^{(2)} - 3x^{(1)} + 2$.
Express the following polynomials in this form:

(a) $25x^2 + 4x - 7$;

(b) $-6x^2 + 14x + 3$.

(c) Show also that any quadratic polynomial with real coefficients can be written in this form.

[So $\{x^{(2)}, x^{(1)}, x^{(0)}\}$ is a basis for the set of polynomials of degree two or less.]

ANSWER

(a) $25x^{(2)} + 29x^{(1)} - 7x^{(0)}$,

(b) $-6x^{(2)} + 8x^{(1)} + 3x^{(0)}$,

(c) $ax^{(2)} + (a+b)x^{(1)} + cx^{(0)}$.