Question
Calculate Log(z) for the following values of z:

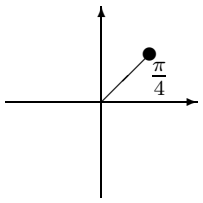
$$(i)z = 0,\ (ii)z = 1 + i,\ (iii)z = i,\ (iv)z = -1 + i$$

$$(v)z = -1.\ (vi)z = -1 - i,\ (vii)z = -i,\ (viii)z = 1 - i$$

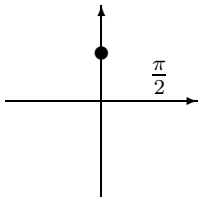
Answer
$\mathrm{Logz} = \log|\mathrm{z}| + \mathrm{iArg(z)} \quad -\pi < \mathrm{Argz} \leq \pi$

(i) $\mathrm{Log0} = \underbrace{\log 0}_{-\infty} + \mathrm{i}\underbrace{\mathrm{Arg}(0)}_{\text{not defined}}$

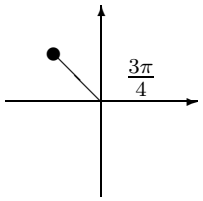
(ii) $\mathrm{Log}(1+\mathrm{i}) = \log|1+\mathrm{i}| + \mathrm{iArg}(1+\mathrm{i}) = \underline{\log\sqrt{2} + \frac{\mathrm{i}\pi}{4}}$



(iii) $\mathrm{Logi} = \log|\mathrm{i}| + \mathrm{iArgi} = 0 + \frac{\mathrm{i}\pi}{2} = \underline{\frac{\mathrm{i}\pi}{2}}$

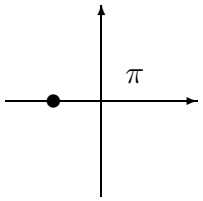


(iv) $\mathrm{Log}(-1+\mathrm{i}) = \log|-1+\mathrm{i}| + \mathrm{i}\mathrm{Arg}(-1+\mathrm{i}) = \underline{\log\sqrt{2} + \mathrm{i}\frac{3\pi}{4}}$



(v) $\mathrm{Log}(-1) = \log|-1| + \mathrm{i}\mathrm{Arg}(-1) = 0 + \mathrm{i}\pi = \underline{\mathrm{i}\pi}$

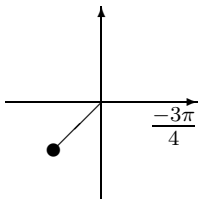
NB $+\pi$ not $-\pi$ since Arg.



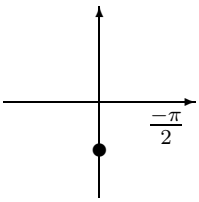
(vi) $\mathrm{Log}(-1-\mathrm{i}) = \log|-1-\mathrm{i}| + \mathrm{i}\mathrm{Arg}(-1-\mathrm{i}) = \log\sqrt{2} + \mathrm{i}\left(-\frac{3\pi}{4}\right) =$

$\underline{\log\sqrt{2} - \frac{3\mathrm{i}\pi}{4}}$

NB $\frac{-3\pi}{4}$ not $\frac{5\pi}{4}$ since $-\pi < \mathrm{Arg} \leq \pi$



(vii) $\mathrm{Log}(-\mathrm{i}) = \log|-\mathrm{i}| + \mathrm{i}\mathrm{Arg}(-\mathrm{i}) = 0 - \dfrac{\mathrm{i}\pi}{2} = \underline{-\dfrac{\mathrm{i}\pi}{2}}$



(viii) $\mathrm{Log}(1-\mathrm{i}) = \log|1-\mathrm{i}| + \mathrm{i}\mathrm{Arg}(1-\mathrm{i}) = \log\sqrt{2} + \mathrm{i}\left(-\dfrac{\pi}{4}\right) = \underline{\log\sqrt{2} - \dfrac{\mathrm{i}\pi}{4}}$