Question

Calculate $\mathbf{u}+\mathbf{v}$, $\mathbf{u}-\mathbf{v}$, $|\mathbf{u}|$, $|\mathbf{v}|$, $\hat{\mathbf{u}}$, $\hat{\mathbf{v}}$, $\mathbf{u}\cdot\mathbf{v}$, the angle between $\mathbf{u}$ and $\mathbf{v}$, the scalar projection of $\mathbf{u}$ in the direction of $\mathbf{v}$, and the vector projection of $\mathbf{u}$ along $\mathbf{v}$ for:

(a) $\mathbf{u}=\mathbf{i}-\mathbf{j},\ \mathbf{v}=2\mathbf{i}+\mathbf{j}+2\mathbf{k}$;

(b) $\mathbf{u}=\mathbf{i}+2\mathbf{k},\ \mathbf{v}=\mathbf{j}+\mathbf{k}$;

(c) $\mathbf{u}=2\mathbf{i}+4\mathbf{j}-3\mathbf{k},\ \mathbf{v}=\mathbf{i}+\mathbf{j}+\mathbf{k}$.

Answer

(a) $\mathbf{u}+\mathbf{v}=3\mathbf{i}+2\mathbf{k},\ \mathbf{u}-\mathbf{v}=-\mathbf{i}-2\mathbf{j}-2\mathbf{k},\ |\mathbf{u}|=\sqrt{2},\ |\mathbf{v}|=3,$

$$\hat{\mathbf{u}}=\frac{1}{\sqrt{2}}\mathbf{i}-\frac{1}{\sqrt{2}}\mathbf{j},\ \hat{\mathbf{v}}=\frac{2}{3}\mathbf{i}+\frac{1}{3}\mathbf{j}+\frac{2}{3}\mathbf{k},\ \mathbf{u}\cdot\mathbf{v}=2-1+0=1,$$

$$\theta=\cos^{-1}\left(\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}||\mathbf{v}|}\right)=\cos^{-1}\left(\frac{1}{3\sqrt{2}}\right)\approx 76.37^\circ,\ \mathbf{u}\cdot\hat{\mathbf{v}}=\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|}=\frac{1}{3},$$

$$(\mathbf{u}\cdot\hat{\mathbf{v}})\hat{\mathbf{v}}=\frac{2}{9}\mathbf{i}+\frac{1}{9}\mathbf{j}+\frac{2}{9}\mathbf{k}.$$

(b) $\mathbf{u}+\mathbf{v}=\mathbf{i}+\mathbf{j}+3\mathbf{k},\ \mathbf{u}-\mathbf{v}=\mathbf{i}-\mathbf{j}+\mathbf{k},\ |\mathbf{u}|=\sqrt{5},\ |\mathbf{v}|=\sqrt{2},$

$$\hat{\mathbf{u}}=\frac{1}{\sqrt{5}}\mathbf{i}+\frac{2}{\sqrt{5}}\mathbf{k},\ \hat{\mathbf{v}}=\frac{1}{\sqrt{2}}\mathbf{j}+\frac{1}{\sqrt{2}}\mathbf{k},\ \mathbf{u}\cdot\mathbf{v}=0+0+2=2,$$

$$\theta=\cos^{-1}\left(\frac{2}{\sqrt{5}\times\sqrt{2}}\right)\approx 50.77^\circ,\ \mathbf{u}\cdot\hat{\mathbf{v}}=\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|}=\frac{2}{\sqrt{2}}=\sqrt{2},$$

$$(\mathbf{u}\cdot\hat{\mathbf{v}})\hat{\mathbf{v}}=\mathbf{j}+\mathbf{k}.$$

(c) $\mathbf{u}+\mathbf{v}=3\mathbf{i}+5\mathbf{j}-2\mathbf{k},\ \mathbf{u}-\mathbf{v}=\mathbf{i}+3\mathbf{j}-4\mathbf{k},\ |\mathbf{u}|=\sqrt{29},\ |\mathbf{v}|=\sqrt{3},$

$$\hat{\mathbf{u}}=\frac{2}{\sqrt{29}}\mathbf{i}+\frac{4}{\sqrt{29}}\mathbf{j}-\frac{3}{\sqrt{29}}\mathbf{k},\ \hat{\mathbf{v}}=\frac{1}{\sqrt{3}}\mathbf{i}+\frac{1}{\sqrt{3}}\mathbf{j}+\frac{1}{\sqrt{3}}\mathbf{k},$$

$$\mathbf{u}\cdot\mathbf{v}=2+4-3=3,\ \theta=\cos^{-1}\left(\frac{3}{\sqrt{29}\times\sqrt{3}}\right)\approx 71.24^\circ,$$

$$\mathbf{u}\cdot\hat{\mathbf{v}}=\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|}=\frac{3}{\sqrt{3}}=\sqrt{3},\ (\mathbf{u}\cdot\hat{\mathbf{v}})\hat{\mathbf{v}}=\mathbf{i}+\mathbf{j}+\mathbf{k}.$$