Exam Question
Topic: Arc Length
Calculate the length of the curve given by

$$y = \ln(\cos x);\ (-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}).$$

Give your answer both in exact form and also as an approximation rounded to four decimal places, using your calculator.

Solution

$$L = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sqrt{1 + f'(x)^2}\, dx,$$

where $f(x) = \ln(\cos x)$
Now $f'(x) = \tan x$ and $1 + \tan^2 x = \sec^2 x$.

$$\begin{aligned} L &= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec x\, dx = [\ln(\sec x + \tan x)]_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \\ &= \ln(\sqrt{2}+1) - \ln(\sqrt{2}-1) = \ln\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right) \\ &= \ln(\sqrt{2}+1)^2 = \ln(3+2\sqrt{2}) = 1.7627(4 \text{ d.p.}) \end{aligned}$$