QUESTION Find the real and imaginary parts $u(x,y)$ and $v(x,y)$ of the function $f(z) = z^3$. Show directly that

(i) u and v satisfy the Cauchy-Riemann equations and

(b) u and v are harmonic functions.

ANSWER $z^3 = ((x+iy)^3 = x^3 - 3xy^2 + i(3x^2y - y^3)$ so that $u(x,y) = x^3 - 3xy^2,\ v(x,y) = 3x^2y - y^3$
Now the ortial derivatives are $u_x = 3x^2 - 3y^2,\ v_y = 3x^2 - 3y^2,\ u_y = -6xy,\ v_x = 6xy,$
so that u and v satisfy the Cauchy-Riemann equations. Also $u_{xx} + v_{yy} = 6x - 6x = 0$ so u is harmonic and similarly v is harmonic.