COMP6037: Foundations of Web Science

The Small World Phenomenon

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Overview

- Definition of Small Worlds
- Results from a social experiment
- The importance of weak ties

Is it Lindsay Lohan or your friends who make you a binge drinker?

via UMBC ebiquity von Tim Finin am 23.06.08

http://arxivblog.com/?p=484
http://arxiv.org/pdf/0806.3176

What determines our behavior or beliefs? Are we influenced by people who are the well-known and popular leaders — political, social, religious — in our society or by the few hundred people that are in our immediate social network — family, friends and co-workers. It's reasonable to assume that it varies by domain or topic, with your music preferences falling in the first category and your spiritual orientation in the second.

Paul Ormerod and Greg Wiltshire have a preprint of a paper 'Binge' drinking in the UK: a social network phenomenon (pdf) that reports on a study that the binge drinking phenomenon seems to spread through "small world" social networks rather than by imitating influentials in a "scale free" network

> "We analyse the recent rapid growth of 'binge' drinking in the UK. This means the consumption of large amounts of alcohol, especially by young people, leading to serious anti-social and criminal behaviour in urban centres. We show how a simple agent-based model, based on binary choice with externalities, combined with a small amount of survey data can explain the phenomenon. We show that the increase in binge drinking is a fashion-related phenomenon, with imitative behaviour spreading across social networks. The results show that a small world network, rather than a random or scale free, offers the best description of the key aspects of the data."

It's fascinating that with the right data, simulation models can help to answer such questions.

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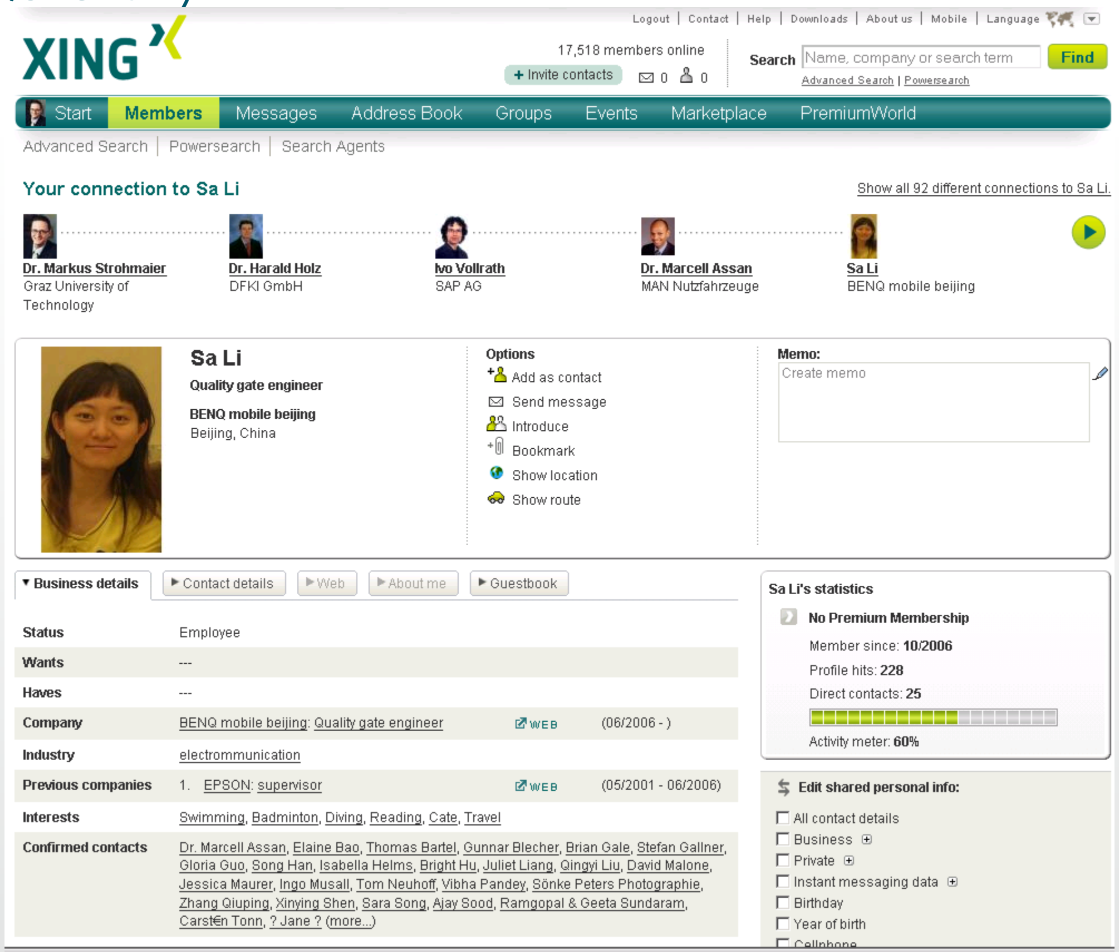
Do I know somebody in …?



The Bacon Number

The Kevin Bacon Game

The oracle of Bacon

www.oracleofbacon.org

The Bacon Number [Watts 2002]

TABLE 3.1 DISTRUBUTION OF ACTORS ACCORDING TO BACON NUMBER

BACON NUMBER	NUMBER OF ACTORS	CUMULATIVE TOTAL NUMBER OF ACTORS
0	1	1
1	1,550	1,551
2	121,661	123,212
3	310,365	433,577
4	71,516	504,733
5	5,314	510,047
6	652	510,699
7	90	510,789
8	38	510,827
9	1	510,828
10	1	510,829

The Erdös Number

Who was Erdös? http://www.oakland.edu/enp/

A famous Hungarian Mathematician, 1913-1996

Erdös posed and solved problems in number theory and other areas and founded the field of discrete mathematics.

- 511 co-authors (Erdös number 1)
- ~ 1500 Publications

The Erdös Number

Through how many research collaboration links is an arbitrary scientist connected to Paul Erdös?

What is a research collaboration link?
- Per definition: Co-authorship on a scientific paper ->
- Convenient: Amenable to computational analysis

What is my Erdös Number? ➔ **5**

me -> W. Hall -> D. Singerman -> G. A. Jones -> P.J. Cameron -> P. Erdös

Stanley Milgram

1933-1984

- A social psychologist
- Yale and Harvard University
- Study on the Small World Problem, beyond well defined communities and relations (such as actors, scientists, ...)
- Controversial: The Obedience Study
- What we will discuss today: *An Experimental Study of the Small World Problem*

Introduction

The simplest way of formulating the small-world problem is: **Starting with any two people in the world, what is the likelihood that they will know each other?**

A somewhat more sophisticated formulation, however, takes account of the fact that while person X and Z may not know each other directly, they may share a mutual acquaintance - that is, a person who knows both of them. One can then think of an acquaintance chain with X knowing Y and Y knowing Z. Moreover, one can imagine circumstances in which X is linked to Z not by a single link, but by a series of links, X-A-B-C-D…Y-Z. That is to say, person X knows person A who in turn knows person B, who knows C… who knows Y, who knows Z.

[Milgram 1967, according to
]http://www.ils.unc.edu/dpr/port/socialnetworking/theory_paper.html#2]

An Experimental Study of the Small World Problem [Travers and Milgram 1969]

A Social Network Experiment tailored towards

- Demonstrating, Defining and Measuring

Inter-connectedness in a large society (USA)

A test of the modern idea of "six degrees of separation"

Which states that: every person on earth is connected to any other person through a chain of acquaintances not longer than 6

Goal

- Define a single target person and a group of starting persons
- Generate an acquaintance chain from each starter to the target

Experimental Set Up

- Each starter receives a document
- was asked to begin moving it by mail toward the target
- Information about the target: name, address, occupation, company, college, year of graduation, wife's name and hometown
- Information about relationship (*friend/acquaintance*) [Granovetter 1973]

Constraints

- starter group was only allowed to send the document to people they know and
- was urged to choose the next recipient in a way as to advance the progress of the document toward the target

Questions

- How many of the starters would be able to establish contact with the target?
- How many intermediaries would be required to link starters with the target?
- What form would the distribution of chain lengths take?

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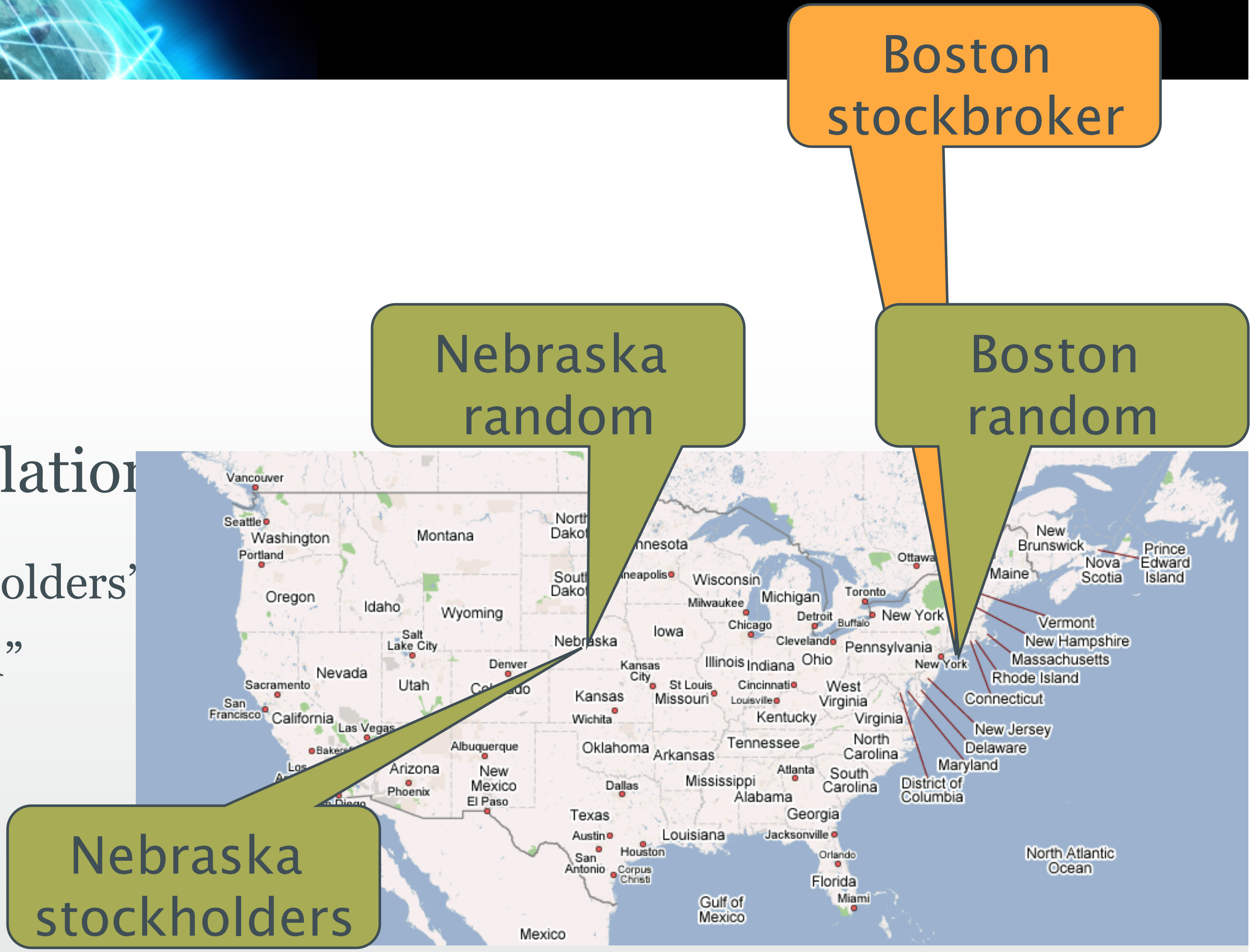
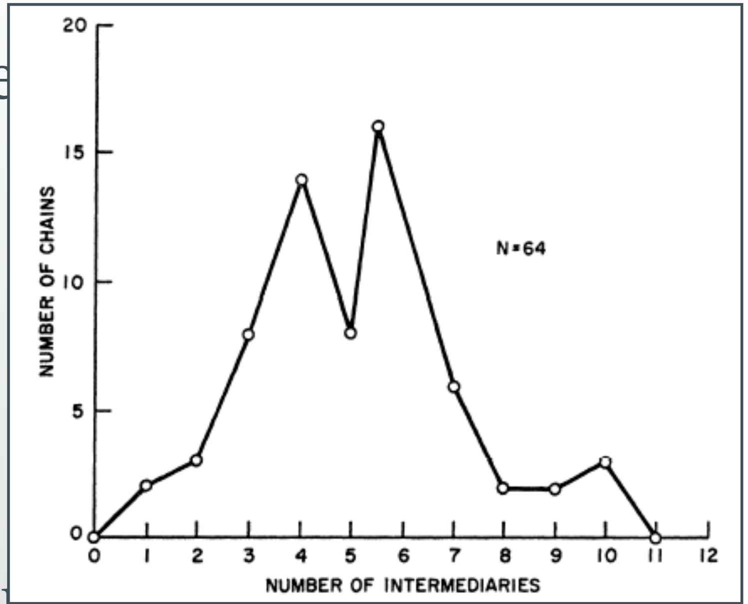
Set Up

- Target person:
  - A Boston stockbroker
- Three starting populations
  - 100 “Nebraska stockholders”
  - 96 “Nebraska random”
  - 100 “Boston random”



Results I

- How many of the starters would be able to establish contact with the target?
  - 64 (or 29%) out of 296 reached the target
- How many intermediaries would be re with the target?
  - Well, that depends: the overall mean 5.2 links
  - Through hometown: 6.1 links
  - Through business: 4.6 links
  - Boston group faster than Nebraska groups
  - Nebraska stakeholders not faster than Nebraska random



Results II

- Incomplete chains

What reasons can you think of for incomplete chains?

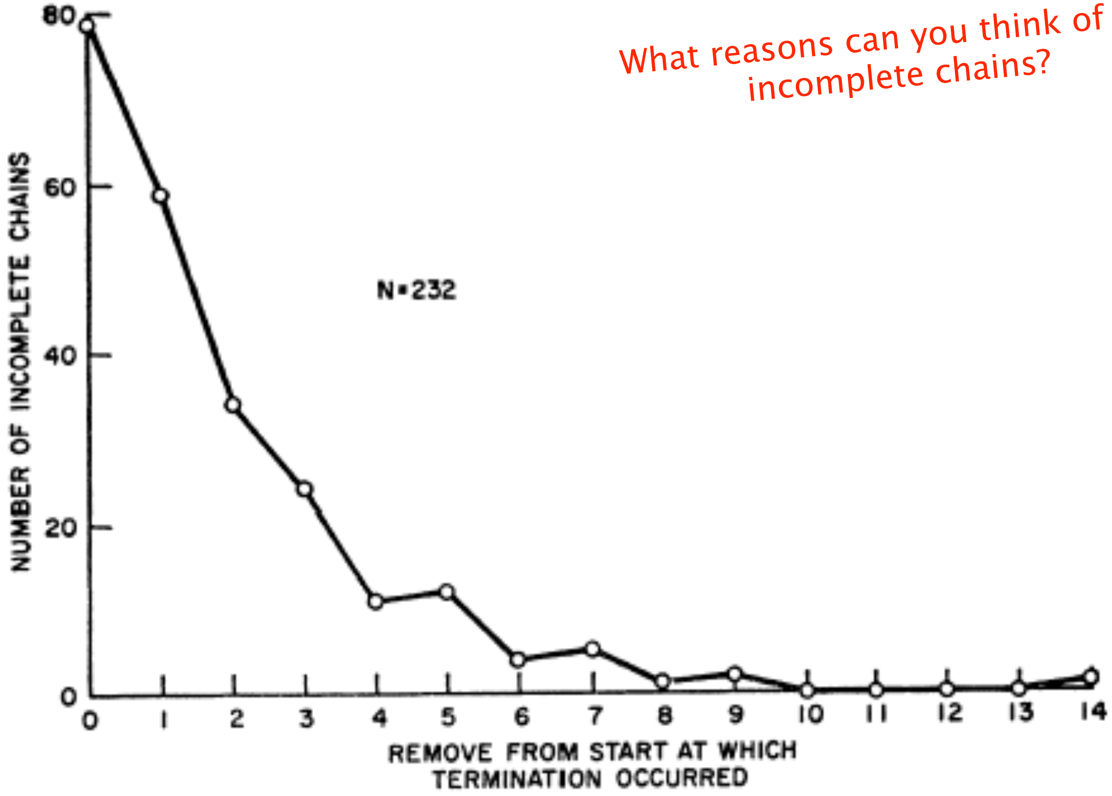


FIGURE 2

Lengths of Incomplete Chains

Results III

- Common paths
- Also see:
  Gladwell's "Law of the few"

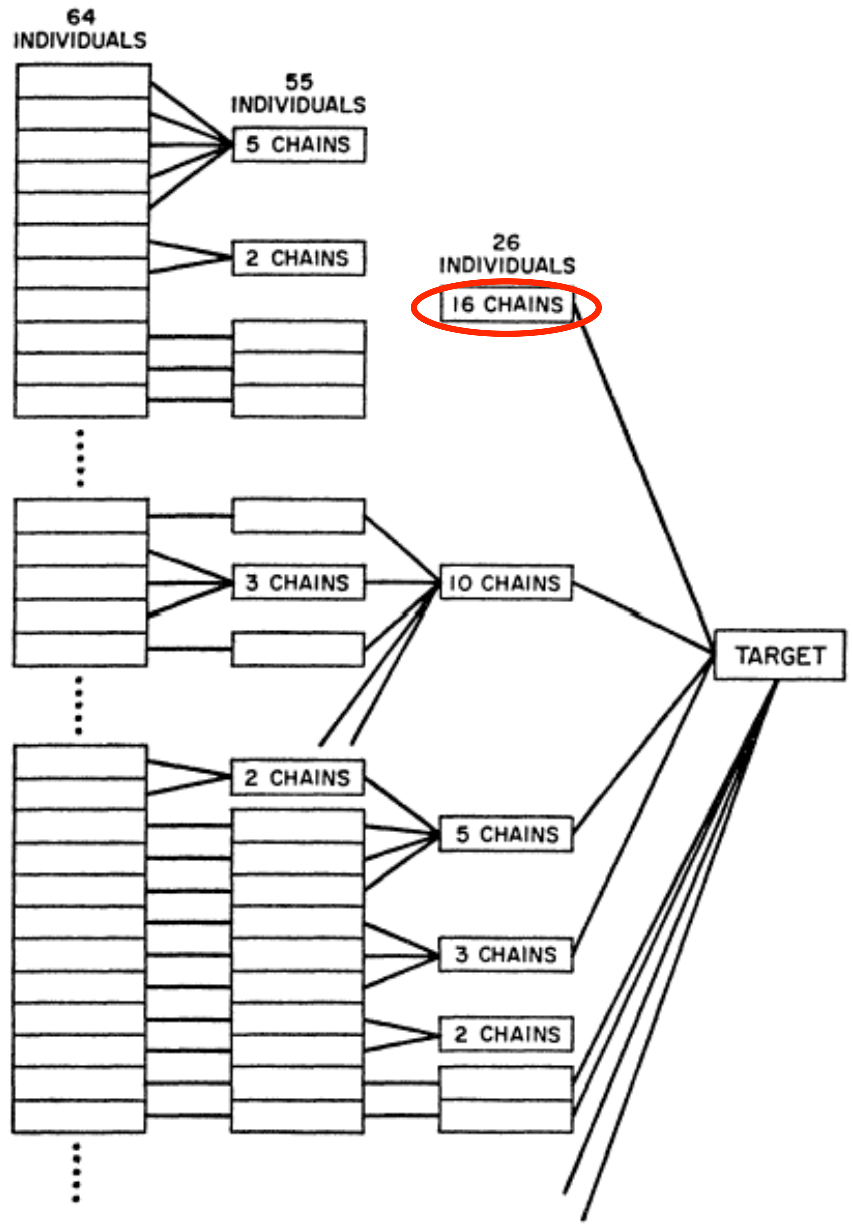


FIGURE 3

Common Paths Appear as Chains Converge on the Target

6 degrees of separation

What kind of problems do you see with the results of this study?

- So is there an upper bound of six degrees of separation in social networks?
  - Extremely hard to test
  - In Milgram's study, ~2/3 of the chains didn't reach the target
  - 1/3 random, 1/3 blue chip owners, 1/3 from Boston
  - Danger of loops (mitigated in Milgram's study through chain records)
  - Target had a "high social status" [Kleinfeld 2000]

Follow up work (2008)

http://arxiv.org/pdf/0803.0939v1

– Horvitz and Leskovec study 2008

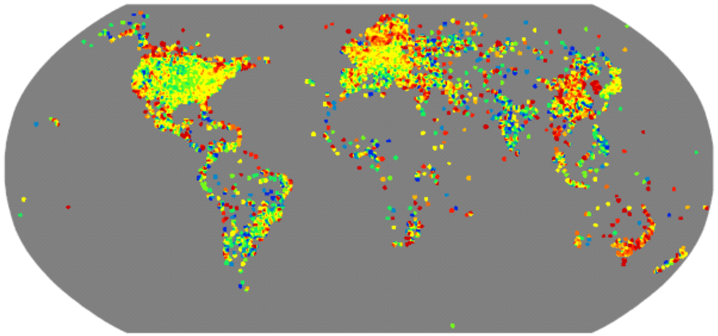
– 30 billion conversations among 240 million people of Microsoft Messenger

– Communication graph with 180 million nodes and 1.3 billion undirected edges

– Largest social network constructed and analyzed to date (2008)

Follow up work (2008)

http://arxiv.org/pdf/0803.0939v1



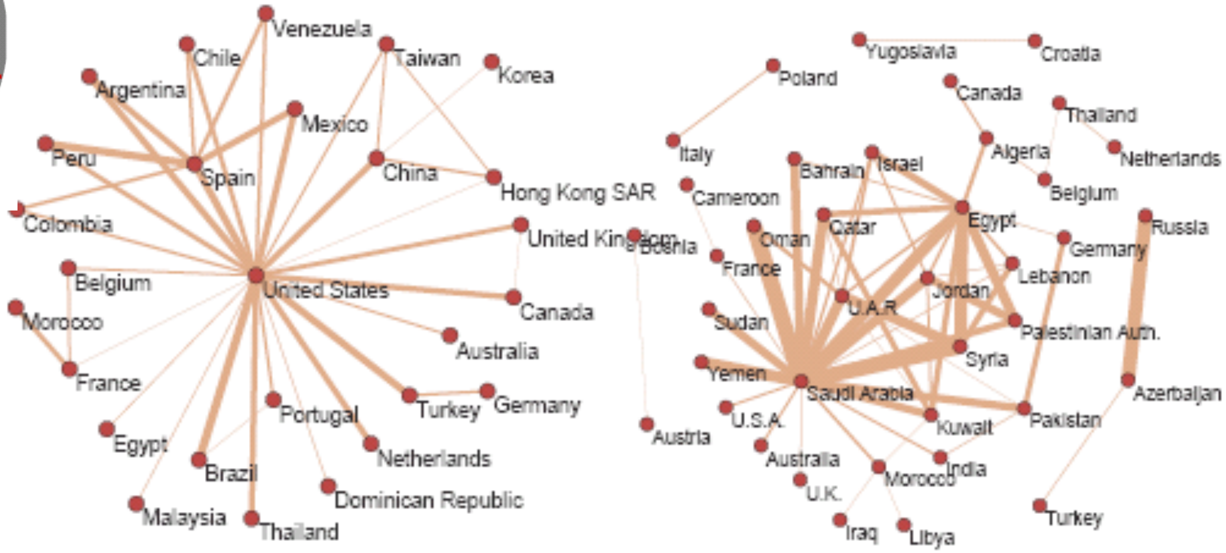
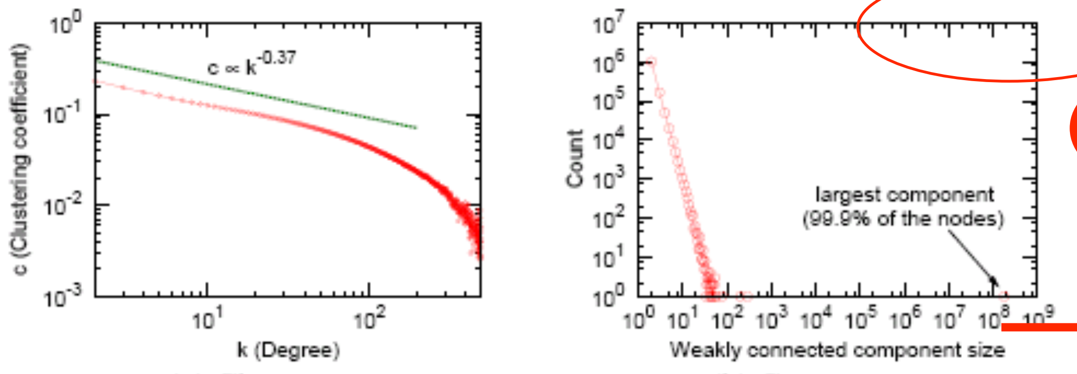


Figure 14: (a) Communication among countries with at least 10 million conversations in June 2006. (b) Countries by average length of the conversation. Edge widths correspond to logarithms of intensity of links.

Follow up work (2008)

http://arxiv.org/pdf/0803.0939v1

- the clustering coefficient decays very slowly with exponent −0.37 with the degree of a node and the average clustering coefficient is 0.137.
- This result suggests that clustering in the Messenger network is much higher than expected—that people with common friends also tend to be connected.



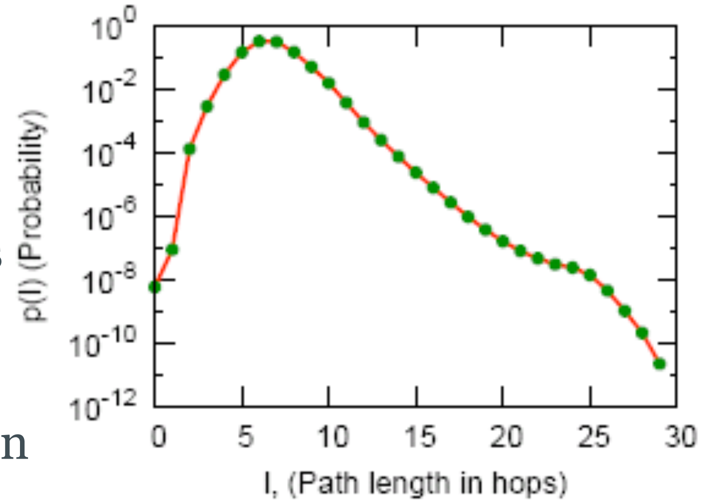
(a) Clustering (b) Components

Figure 19: (a) Clustering coefficient; (b) distribution of connected components. 99.9% of the nodes belong to the largest connected component.

Follow up work (2008)

Approximation of "Degrees of separation"

- Random sample of 1000 nodes
- for each node the shortest paths to all other nodes was calculated. The average path length is 6.6. median at 7.
- Result: a random pair of nodes is 6.6 hops apart on the average, which is half a link longer than the length reported by Travers/Milgram.
- The 90th percentile (effective diameter (16)) of the distribution is 7.8. 48% of nodes can be reached within 6 hops and 78% within 7 hops.
- we find that there are about "7 degrees of separation" among people.
- long paths exist in the network; we found paths up to a length of 29.



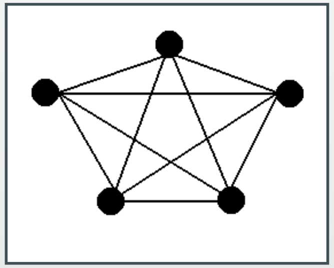
(a) Diameter

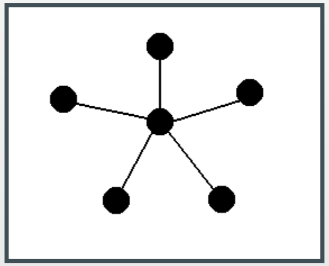
Small Worlds

http://www.infosci.cornell.edu/courses/info204/2007sp/

- Every pair of nodes in a graph is connected by a path with an extremely small number of steps
(low diameter)

- Two principle ways of encountering small worlds
  - Dense networks
  - sparse networks with well-placed connectors





Example for base e

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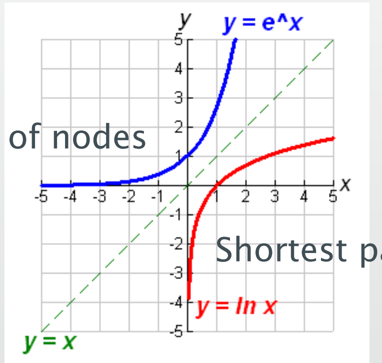
Small Worlds [Newman 2003]

- The small-world effect exists, if
  - The number of vertices within a distance r of a typical central vertex grows exponentially with r (the larger it get, the faster it grows) $x(t) = x_0 e^{kt}$

In other words:

  - Networks are said to show the small-world effect if the value of l (avg. shortest distance) scales logarithmically or slower with network size for fixed mean degree $e^{\ln(x)} = x \quad \text{if } x > 0$

Number of nodes



Shortest path

Contemporary Software

- Where does the small-world phenomenon come into play in contemporary software, in organizations, ..?

How do Small World Networks form?

- Xing, LinkedIn, Myspace, Facebook, FOAF, ...
- Business Processes, Information and Knowledge Flow

Preferential Attachment [Barabasi 1999]

The rich getting richer

Preferential Attachment refers to the high probability of a new vertex to connect to a vertex that already has a large number of connections

1. a new website linking to more established ones
2. a new individual linking to well-known individuals in a social network

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Preferential Attachment Example

Which node has the highest probability of being linked by a new node in a network that exhibits traits of preferential attachment?

Example

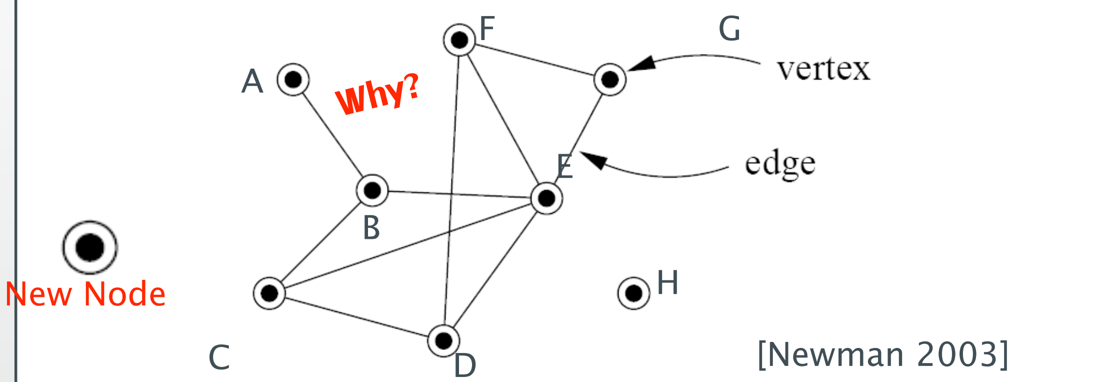


FIG. 1 A small example network with eight vertices and ten edges.

Assortative Mixing (or Homophily) [Newman 2003]

Assortative Mixing refers to selective linking of nodes to other nodes who share some common property

- E.g. degree correlation
  high degree nodes in a network associate preferentially with other high-degree nodes

- E.g. social networks
  nodes of a certain type tend to associate with the same type of nodes (e.g. by race)

Assortative Mixing (or Homophily) [Newman 2003]

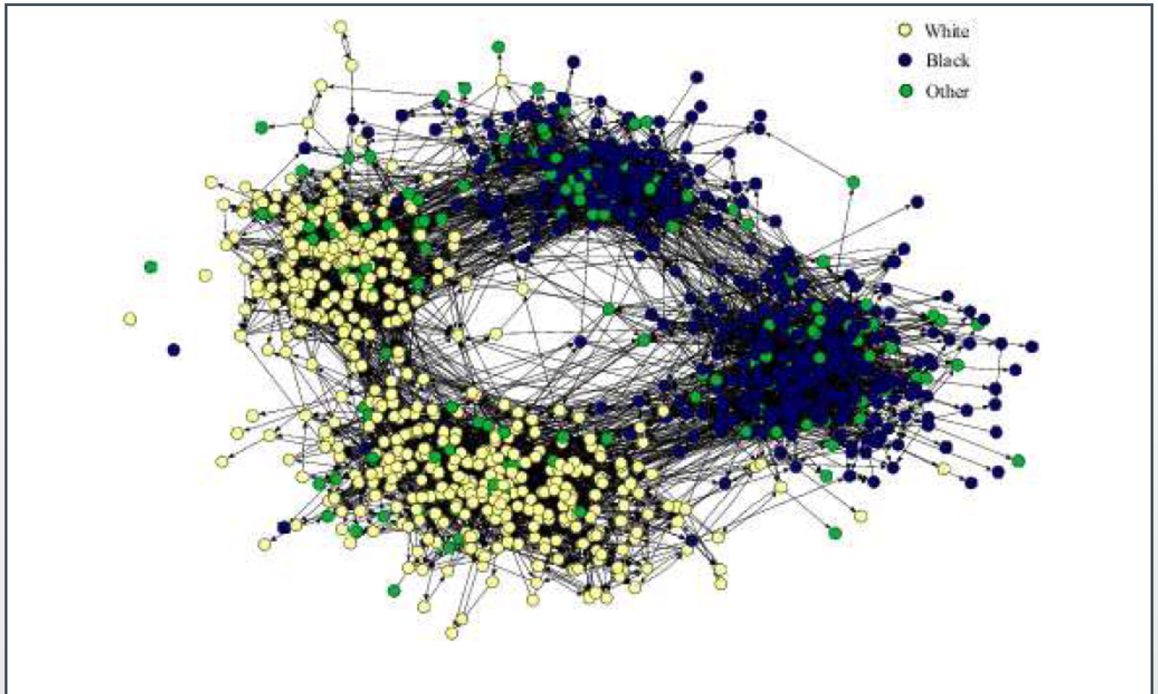


FIG. 8 Friendship network of children in a US school. Friendships are determined by asking the participants, and hence are directed, since A may say that B is their friend but not *vice versa*. Vertices are color coded according to race, as marked, and the split from left to right in the figure is clearly primarily along lines of race. The split from top to bottom is between middle school and high school, i.e., between younger and older children. Picture courtesy of James Moody.

Disassortativity [Newman 2003]

Disassortativity refers to selective linking of nodes to other nodes who are different in some property

- E.g. the web
  low degree nodes tend to associate with high degree nodes

Network Resilience [Newman 2003]

The resilience of networks with respect to vertex removal and network connectivity.

If vertices are removed from a network, the typical length of paths between pairs of vertices will increase – vertex pairs will be disconnected.

Examples:

1. Deletion of a hub
2. Deletion of a leaf node element

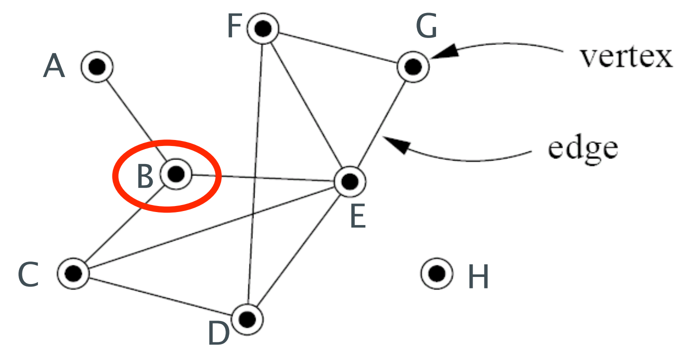
Network Resilience [Newman 2003]

Connectivity: a function of whether a graph remains connected when nodes and/or lines are deleted. [Wassermann 1994]

Delete the node with the highest degree, what happens to the network?

Deleting which nodes introduces a new component?

Example



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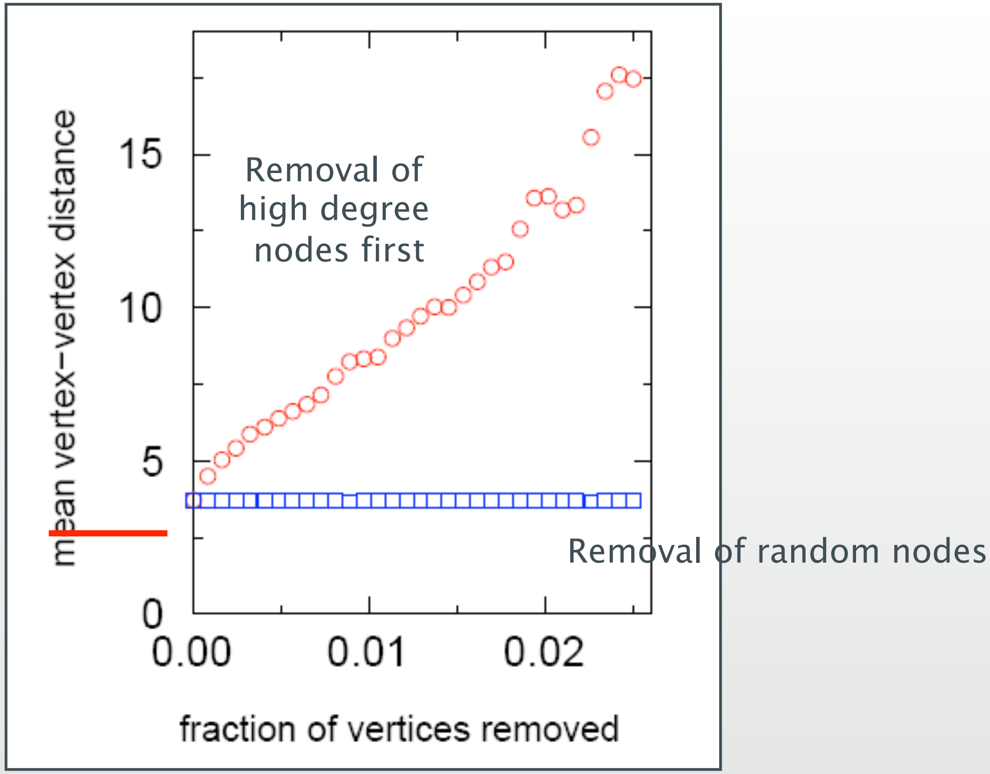
Network Resilience [Newman 2003]



Connectivity of the Web [Newman 2003, Broder et al 2000]

What does it need to destroy the connectivity of the web?

According to Broder et al 2000, you need to remove all vertices with a degree greater than five.

Because of the highly skewed degree distribution of the web, the fraction of vertices with degree greater than five is only a small fraction of all vertices.

But …

Isn‘t all of this an over simplification of the world of social systems?

– Ties/relationships vary in intensity

– People who have strong ties tend to share a similiar set of acquaintances

– Ties change over time

– Nodes (people) have different characteristics, and they are *actors*

The Strength of Weak Ties [Granovetter 1973]

The strength of an interpersonal tie is a

- (probably linear) combination of the amount of time
- The emotional intensity
- The intimacy
- The reciprocal services which characterize the tie

Mark Granovetter, Stanford University

Can you give examples of strong / weak ties?

The Strength of Weak Ties and Mutual Acquaintances [Granovetter 1973]

Consider:

Two arbitrarily selected individuals A and B and

The set S = C,D,E of all persons with ties to either or both of them

Hypothesis:

The stronger the tie between A and B, the larger the proportion of individuals in S to whom they will both be tied.

Theoretical corroboration:

Stronger ties involve larger time commitments – probability of B meeting with some friend of A (who B does not know yet) is increased

The stronger a tie connecting two individuals, the more similar they are

The Strength of Weak Ties [Granovetter 1973]

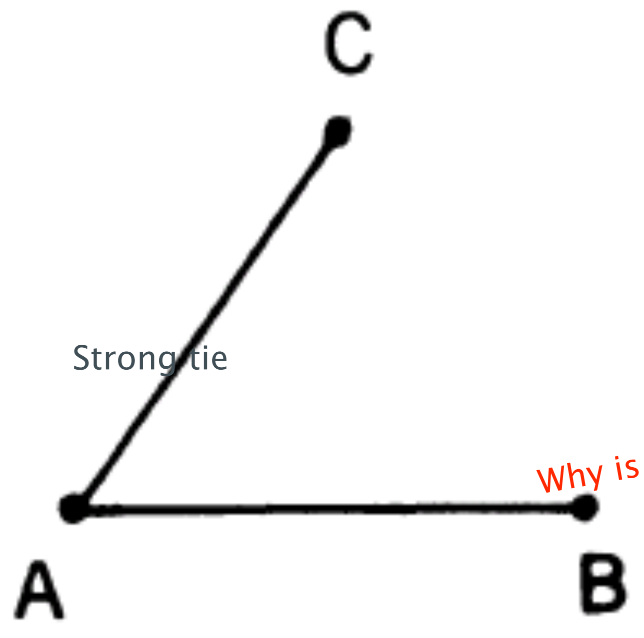


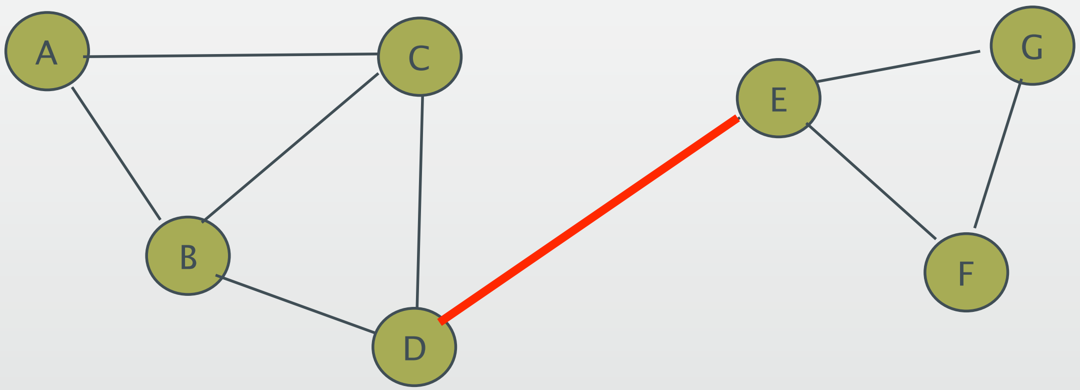
FIG. 1.—Forbidden triad

Why is it called the forbidden triad?

Bridges [Granovetter 1973]

A bridge is a line in a network which provides **the only path** between two points.

In social networks, a bridge between A and B provides the only route along which information or influence can flow from any contact of A to any contact of B



Which edge represents a bridge? Why?

Bridges and Strong Ties [Granovetter 1973]

Example:

1. Imagine the strong tie between A and B
2. Imagine the strong tie between A and C
3. Then, the forbidden triad **implies** that a tie **exists** between C and B

   (it forbids that a tie between C and B does not exist)

1. From that follows, that A-B is not a bridge (because there is another path A-B that goes through C)

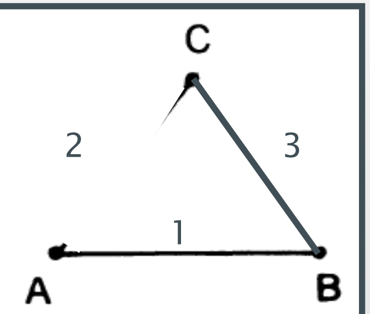


FIG. 1.—Forbidden triad

Why is this interesting?

⇨Strong ties can be a bridge ONLY IF neither party to it has any other strong ties

⇨Highly unlikely in a social network of any size

⇨Weak ties suffer no such restriction, though they are not automatically bridges

⇨But, all bridges are weak ties

In Reality .... [Granovetter 1973]

it probably happens only rarely, that a specific tie provides the only path between two points

Bridges are efficient paths

- Alternatives are more costly
- Local bridges of degree n
- A local bridge is more significant as its degree increases

Local bridges: the shortest path between its two points (other than itself)

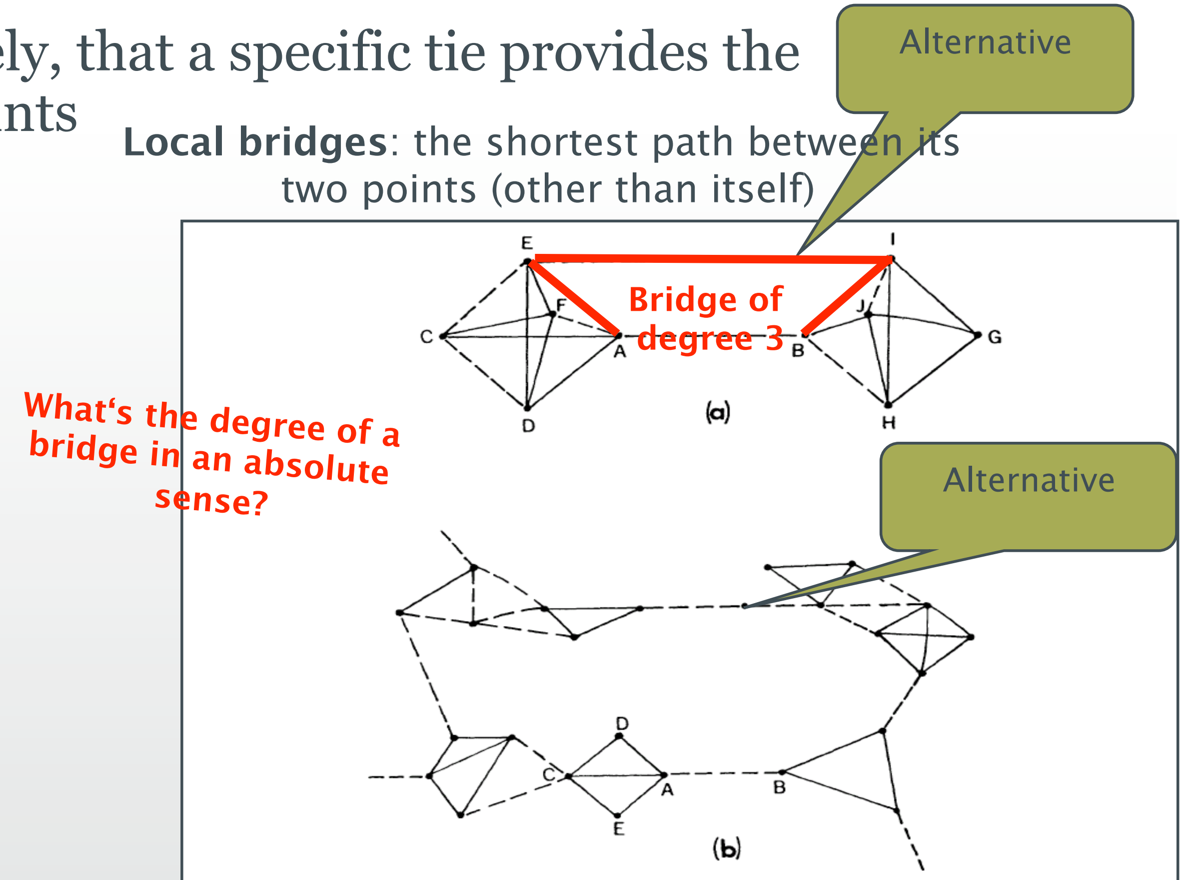


FIG. 2.—Local bridges. *a*, Degree 3; *b*, Degree 13. ——— = strong tie; – – – = weak tie.

In Reality ...

Strong ties can represent *local* bridges BUT

They are weak (i.e. they have a low degree)

What's the degree of the local bridge A-B?

Why?

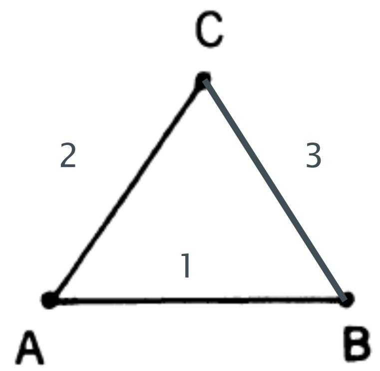


FIG. 1.—Forbidden triad

Implications of Weak Ties [Granovetter 1973]

Those weak ties, that are local bridges, create more, and shorter paths.

– The removal of the average weak tie would do more damage to transmission probabilities than would that of the average strong one

– **Paradox**: While *weak ties* have been denounced as generative of alienation, *strong ties*, breeding local cohesion, lead to overall fragmentation

What are sources of weak ties/bridges?

Can you identify some implications for social networks on the web / for search in these networks?

How does this relate to Milgram's experiment?

Completion rates in Milgram's experiment were reported higher for acquaintance than friend relationships [Granovetter 1973]

Implications of Weak Ties [Granovetter 1973]

- Example: Spread of information/rumors in social networks
  - Studies have shown that people rarely act on mass-media information unless it is also transmitted through personal ties [Granovetter 2003, p 1274]
  - Information/rumors moving through strong ties is much more likely to be limited to a few cliques than that going via weak ones, bridges will not be crossed

How does information spread through weak ties?